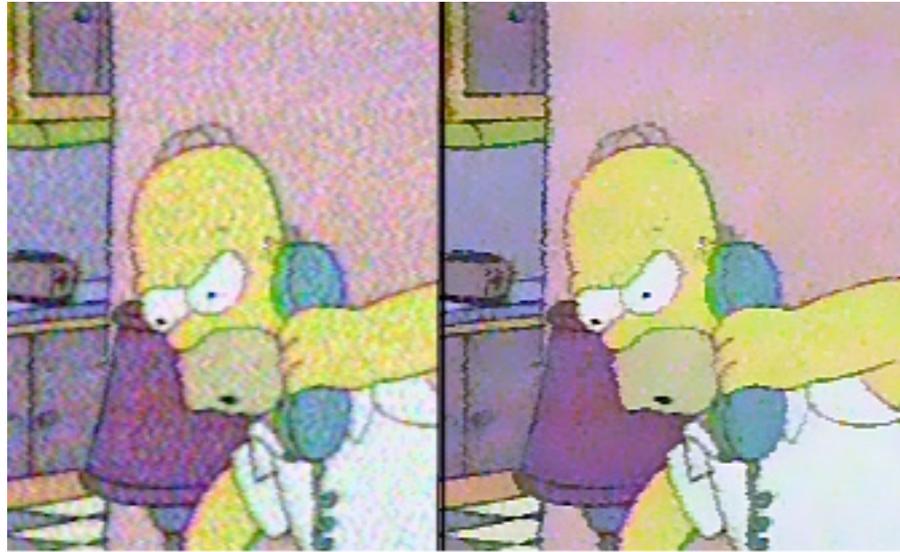


# Schärfe durch Unschärfe

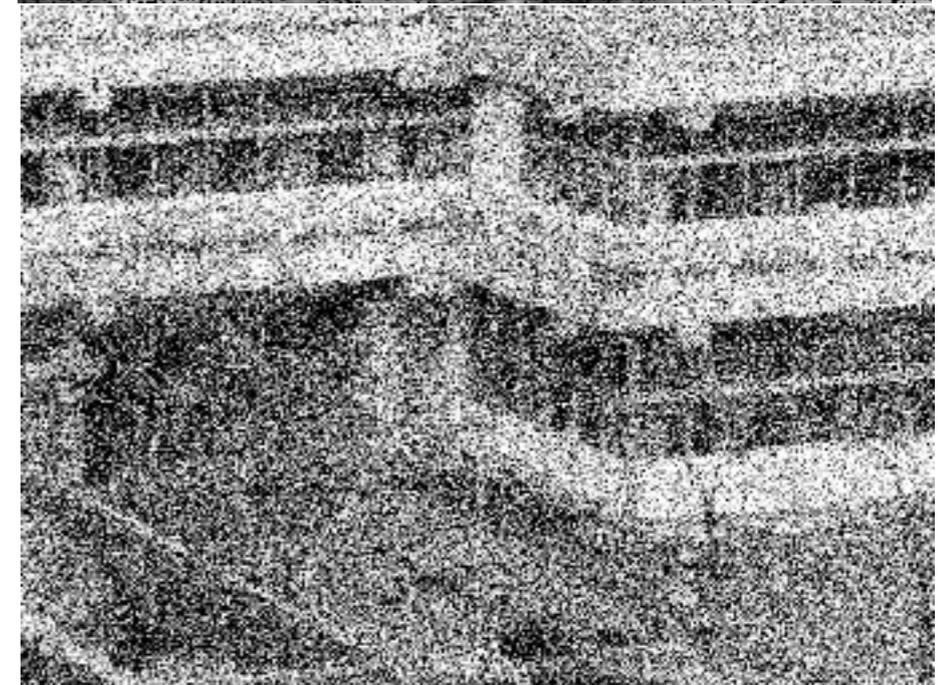
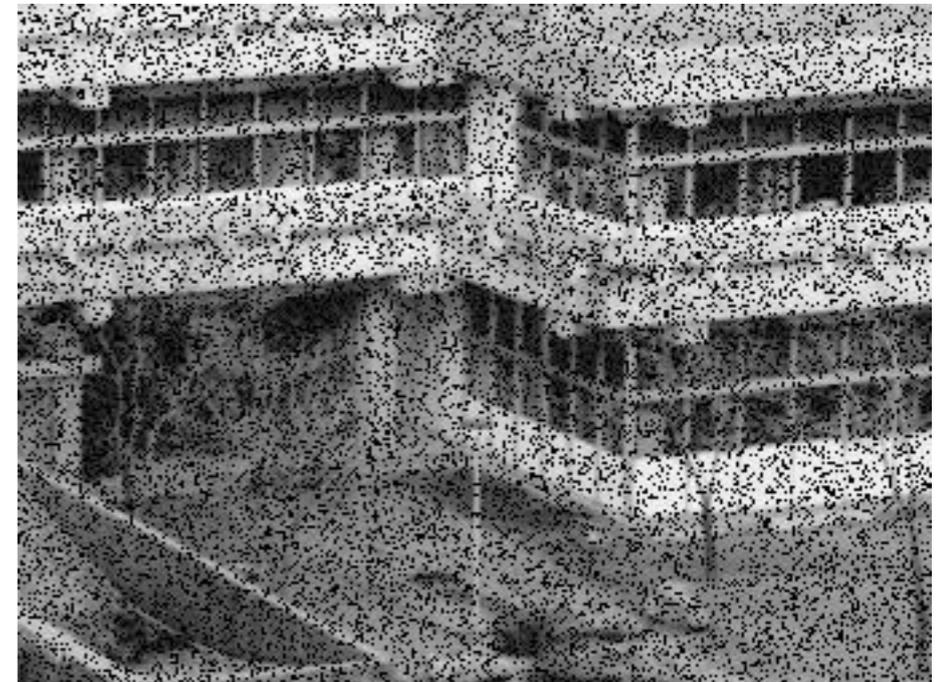
Phänomene an der Grenze zwischen  
Analog und Digital

Dr. Jochen Koubek  
Humboldt-Universität zu Berlin

Hyperkult XIII – Unschärfe



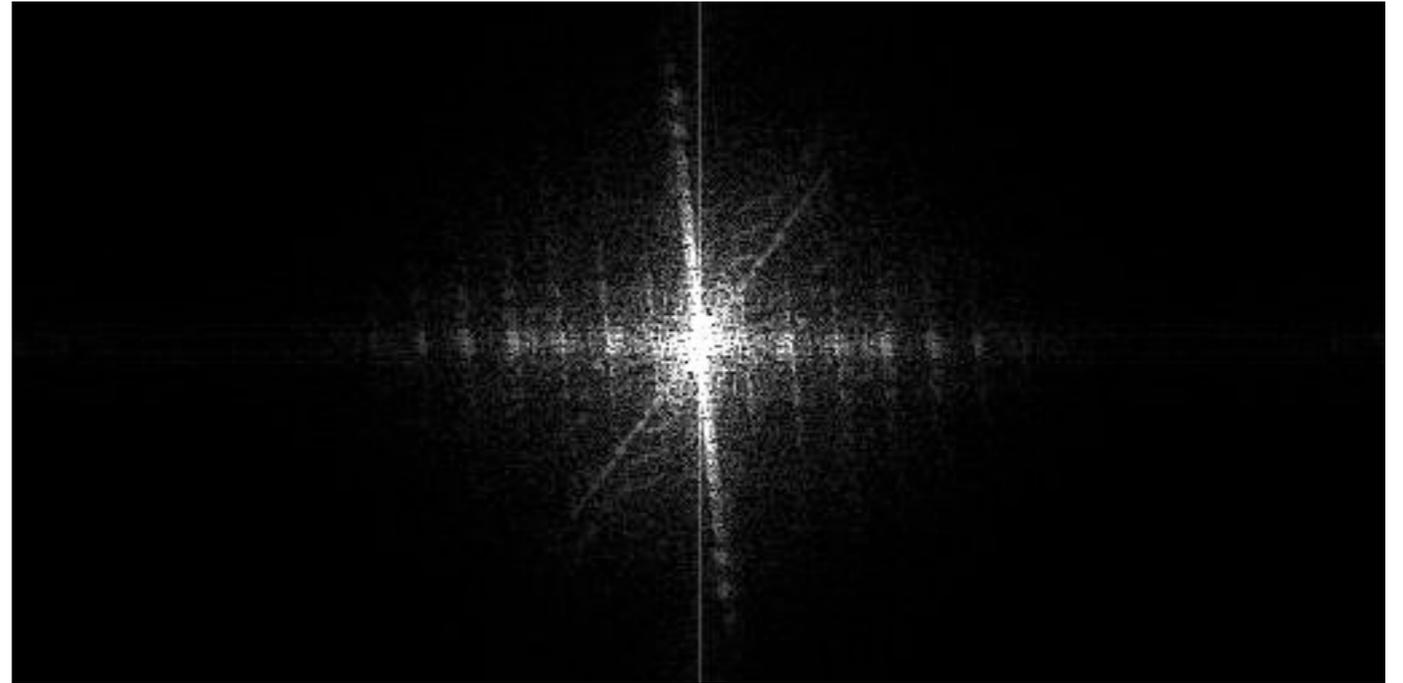
# Rauschen



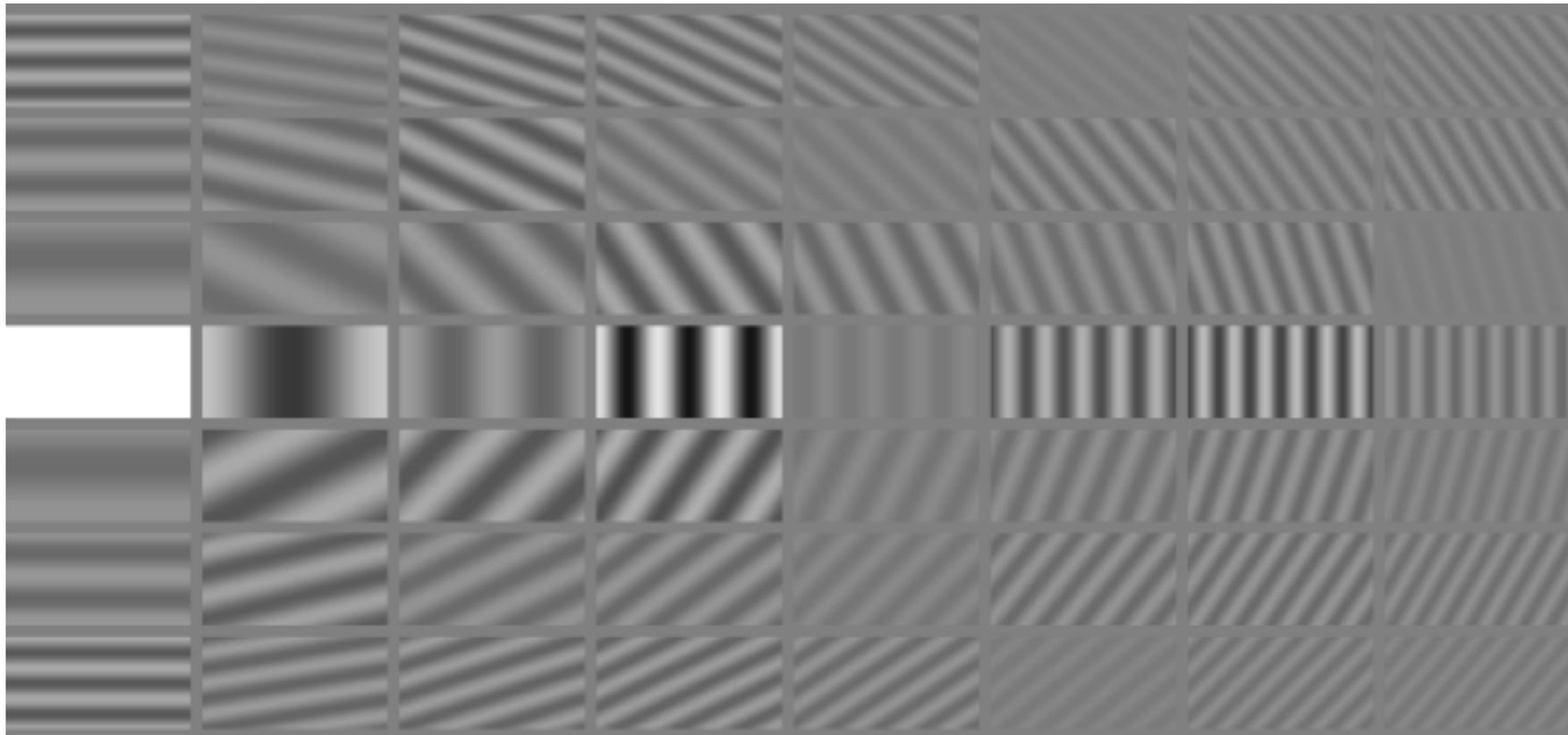
# 2D Fourier-Transformation



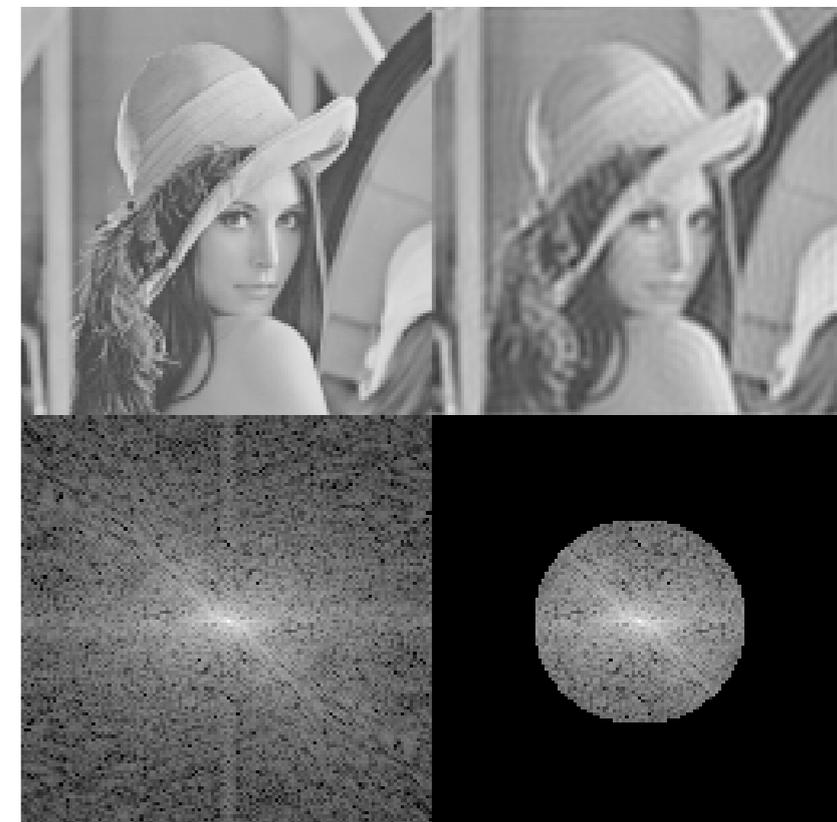
Ausgangsbild



Fouriertransformierte



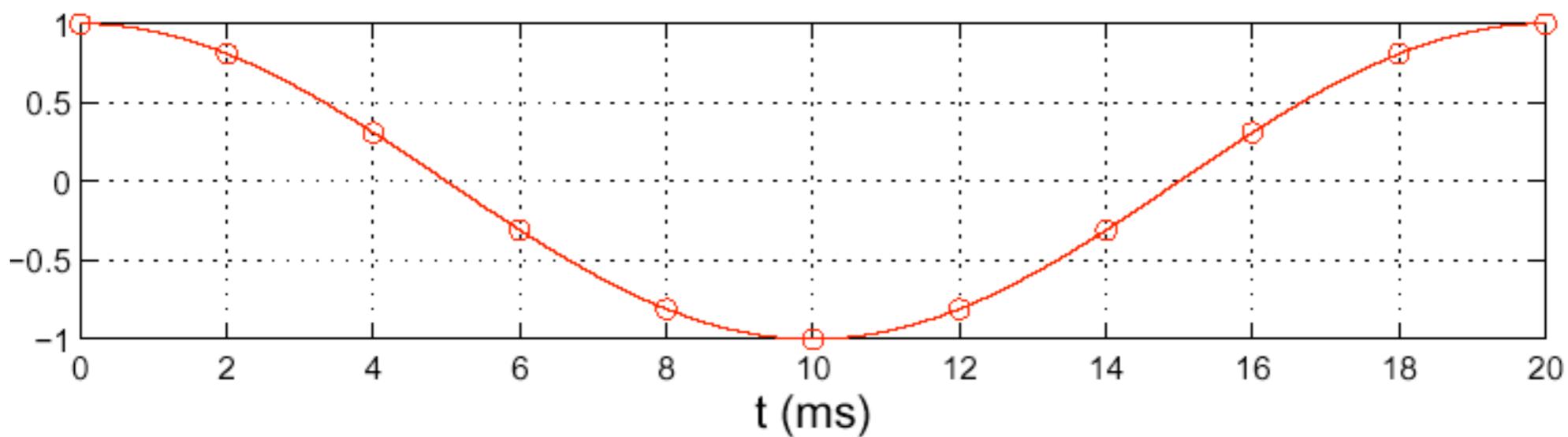
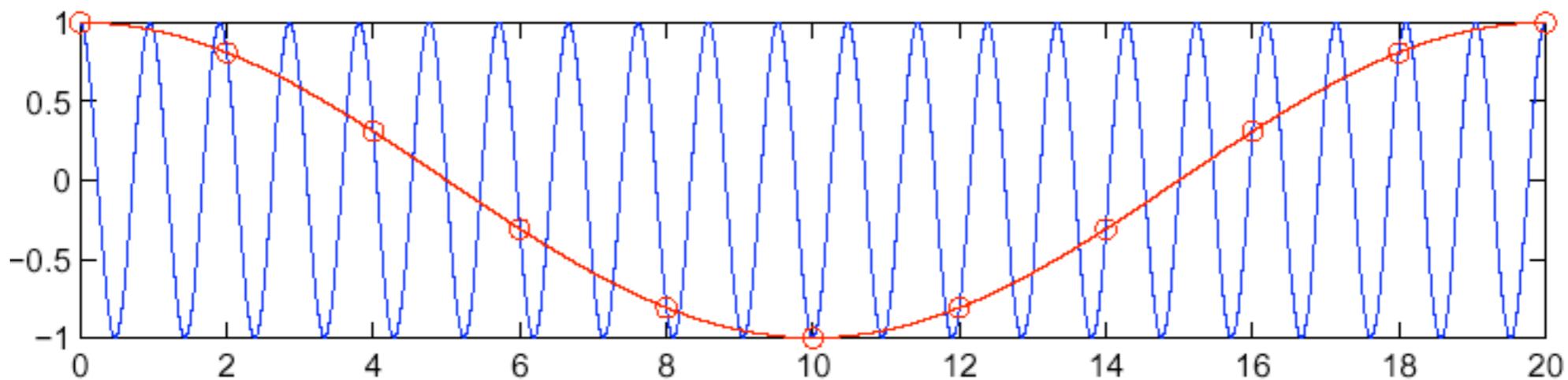
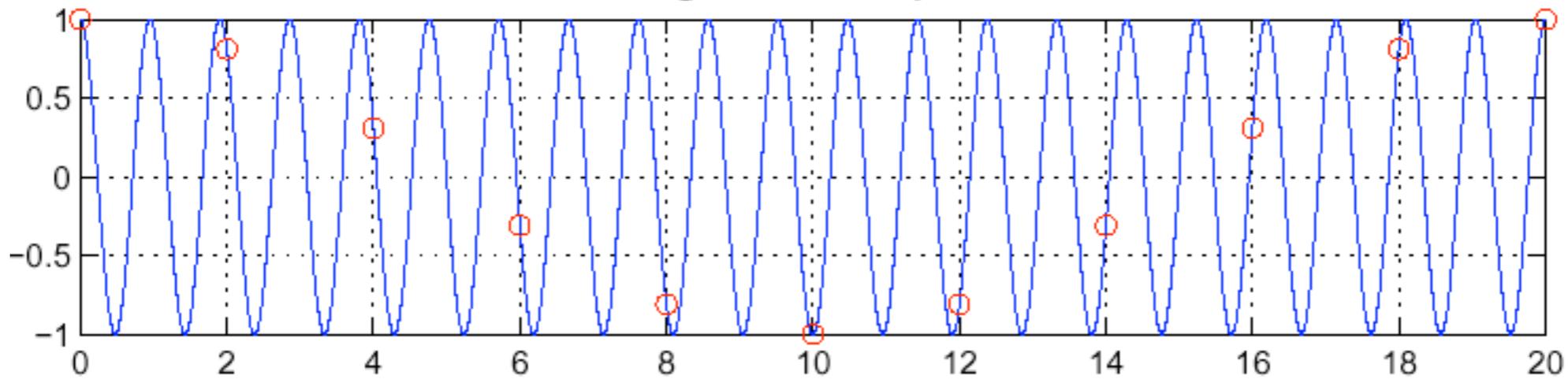
Basisbilder der Fouriertransformation



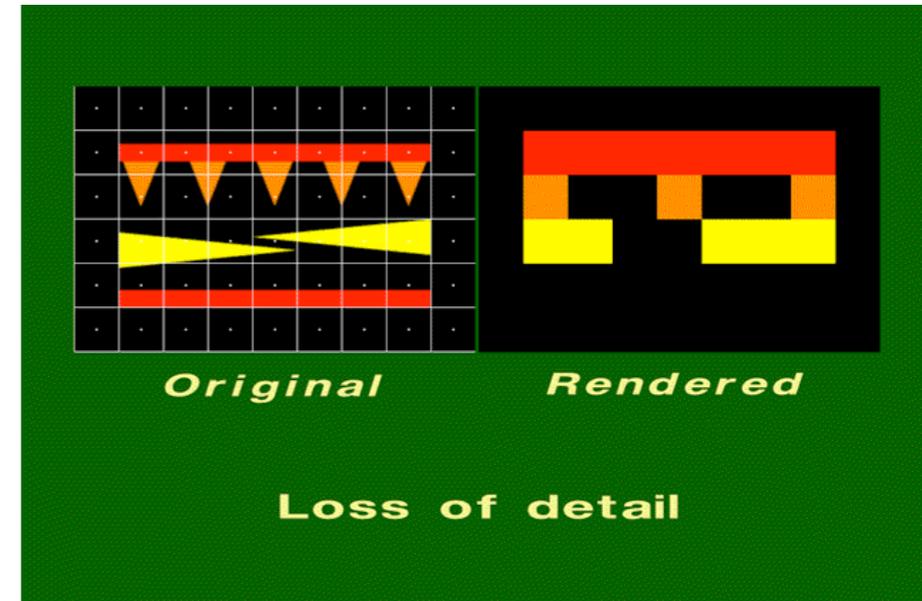
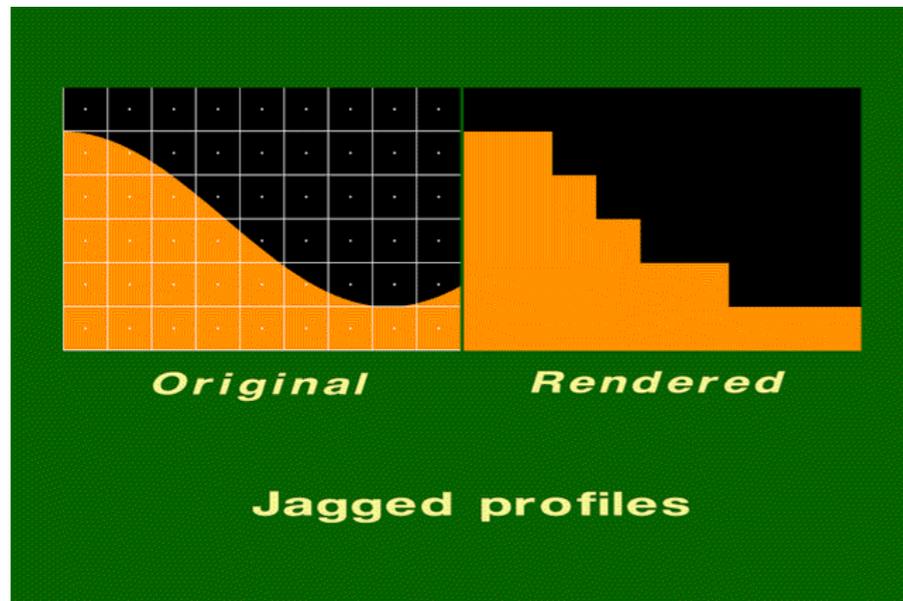
Tiefpassfilter

# Nyquisttheorem

A 1050 Hz signal sampled at 500 Hz



# Aliasing

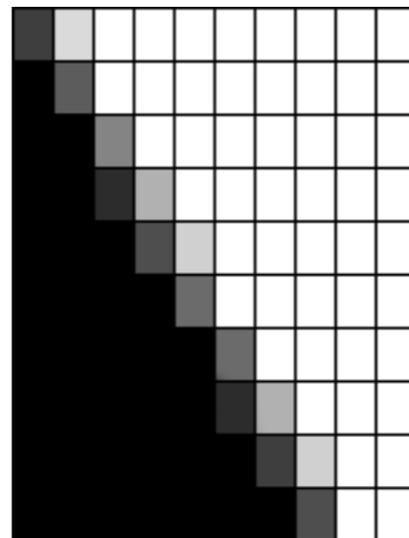
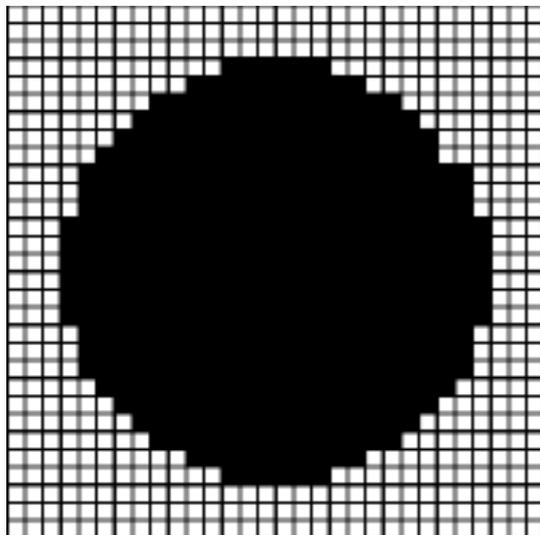
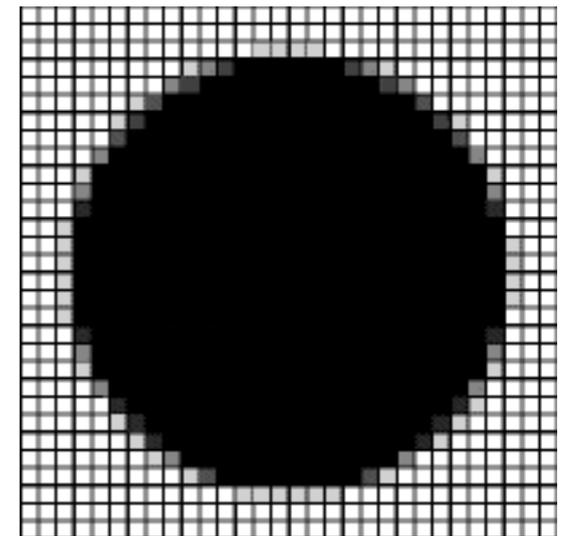
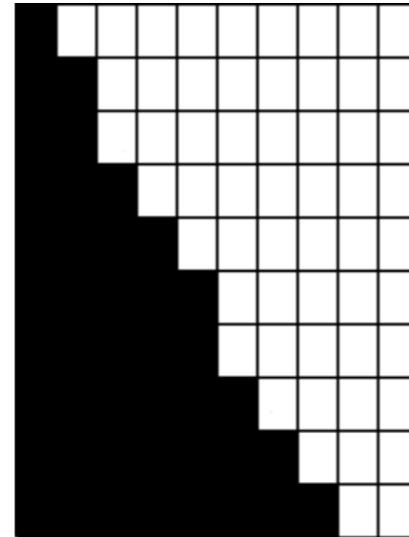
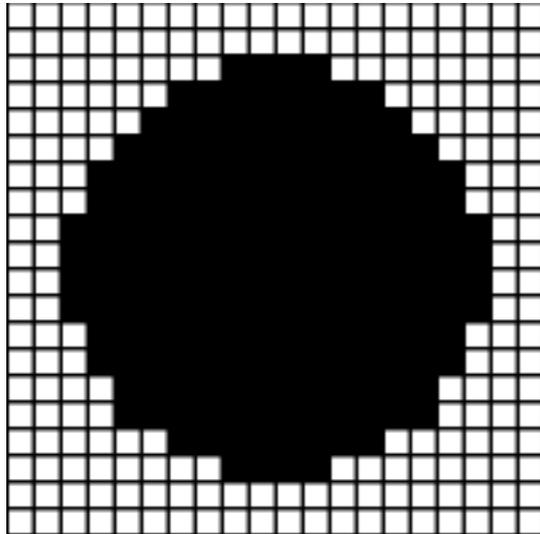


**Mathematiker:** Doppelte Frequenzantwort

**Nachrichtentechniker:** Abtasten unterhalb der Nyquist-Rate

**Programmierer:** Treppen

# Anti-Aliasing



Auflösung des  
Pixelrasters erhöhen

Grauschattierungen

# Area Sampling

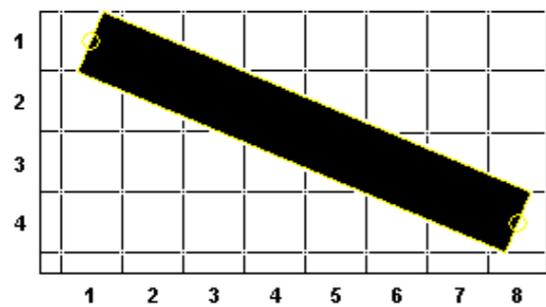
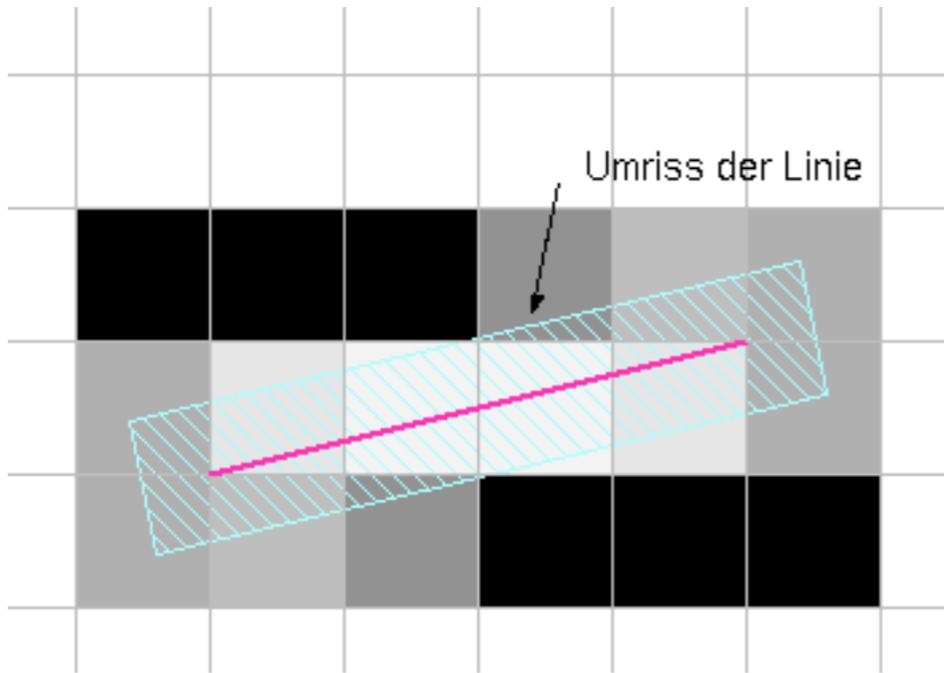


Figure A: unit width line

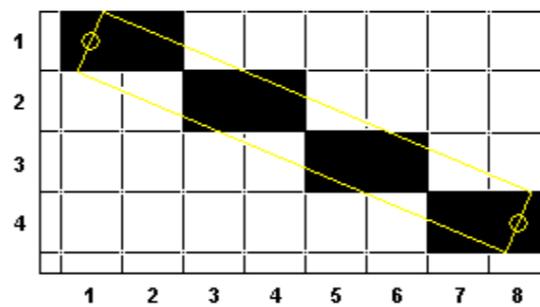


Figure B:  $y=f(x)$  approximation

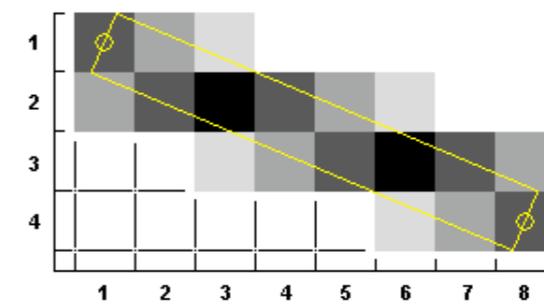
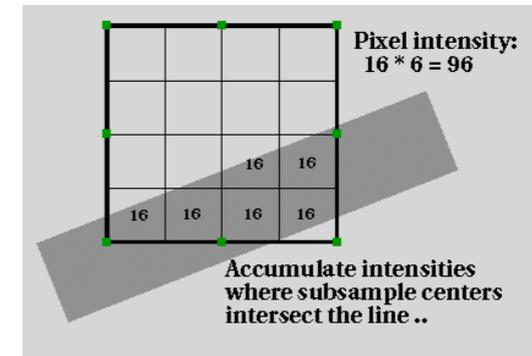
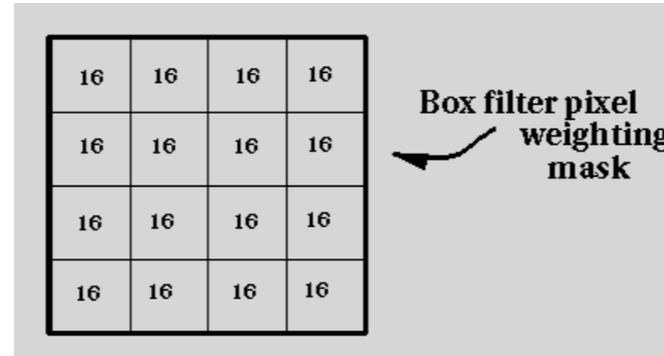
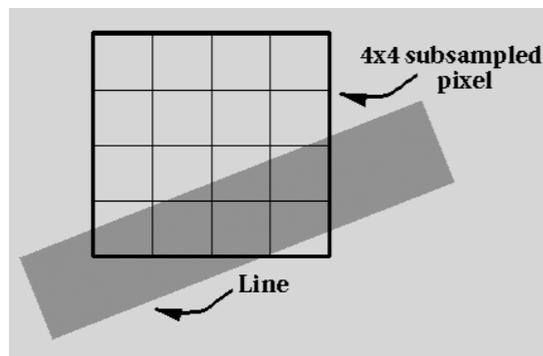
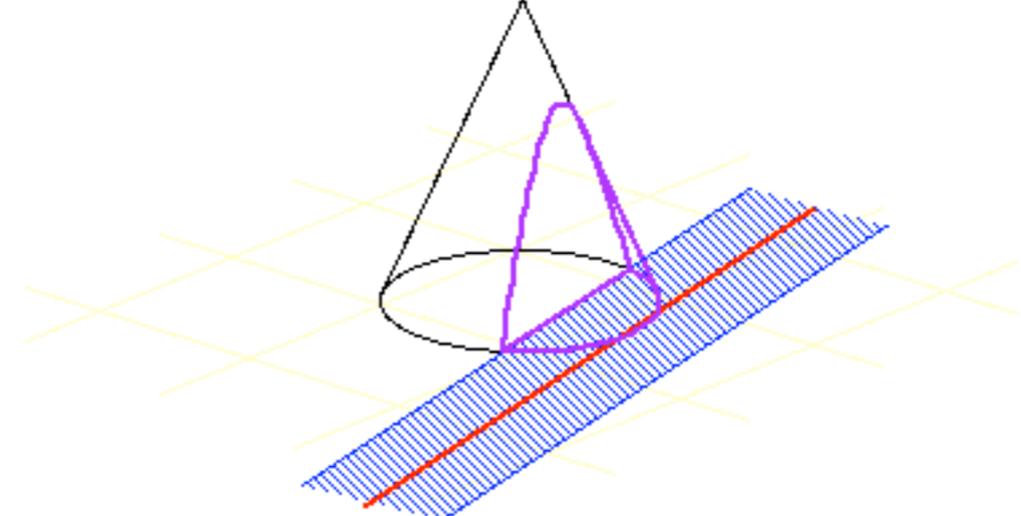
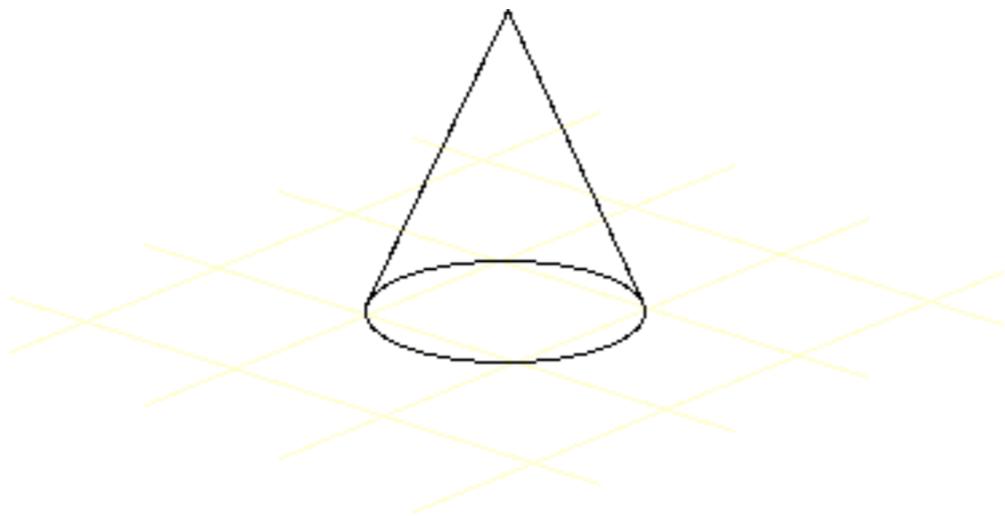
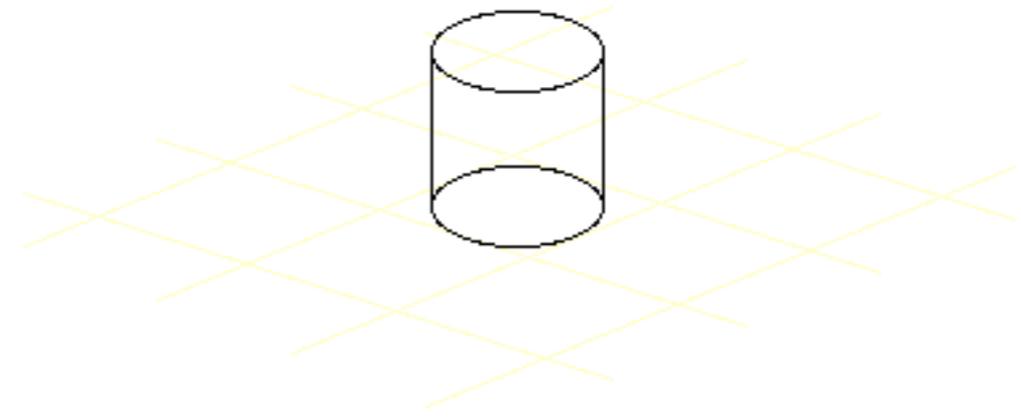
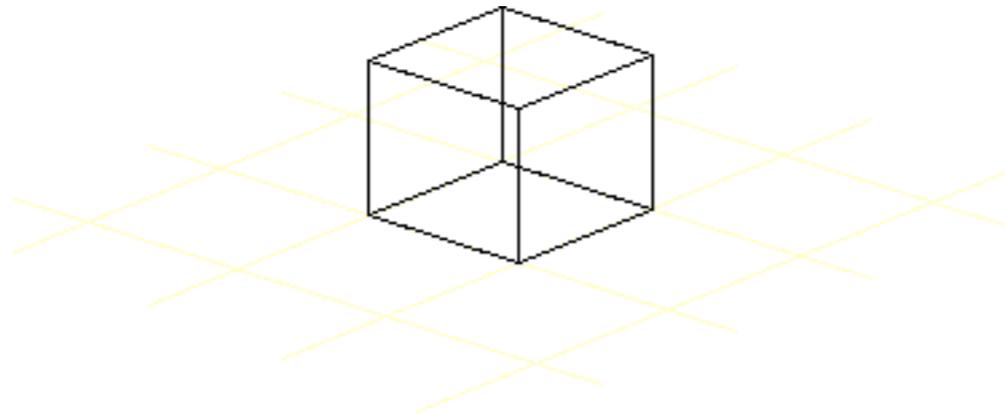
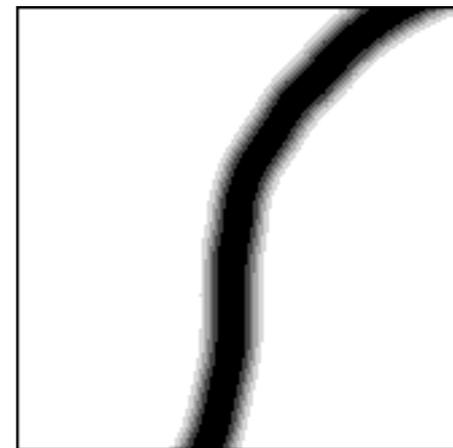
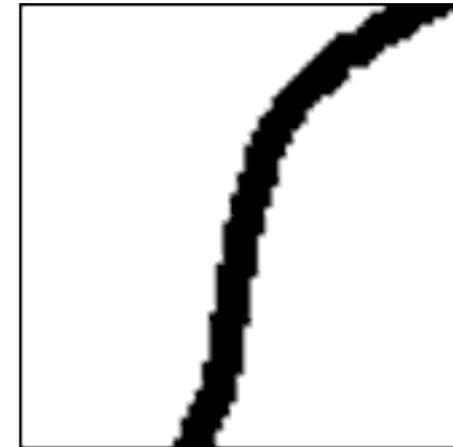
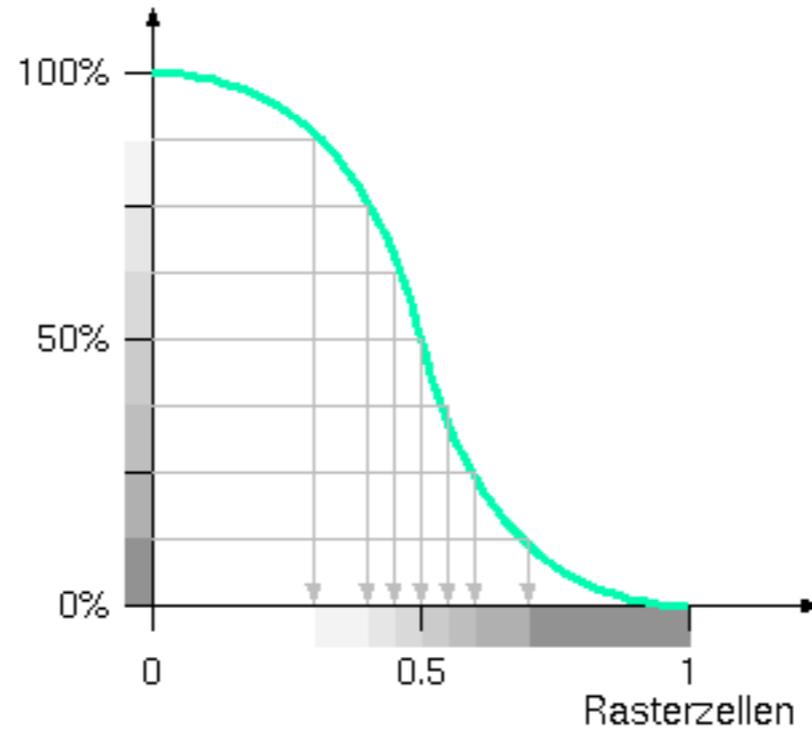
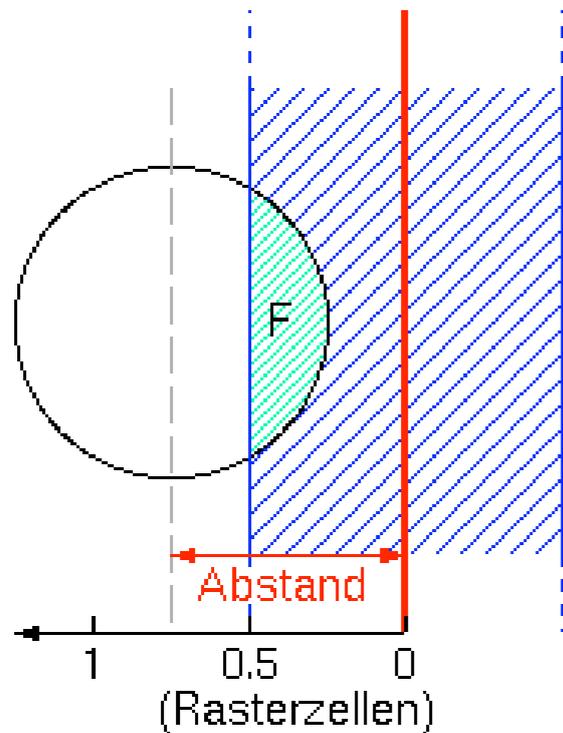


Figure C: Pixel Color by Coverage

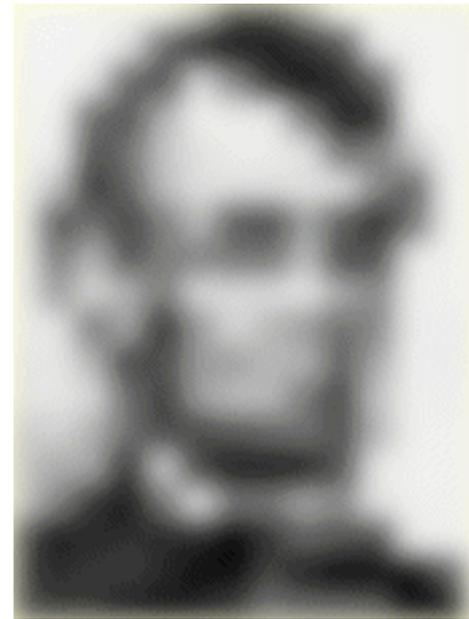
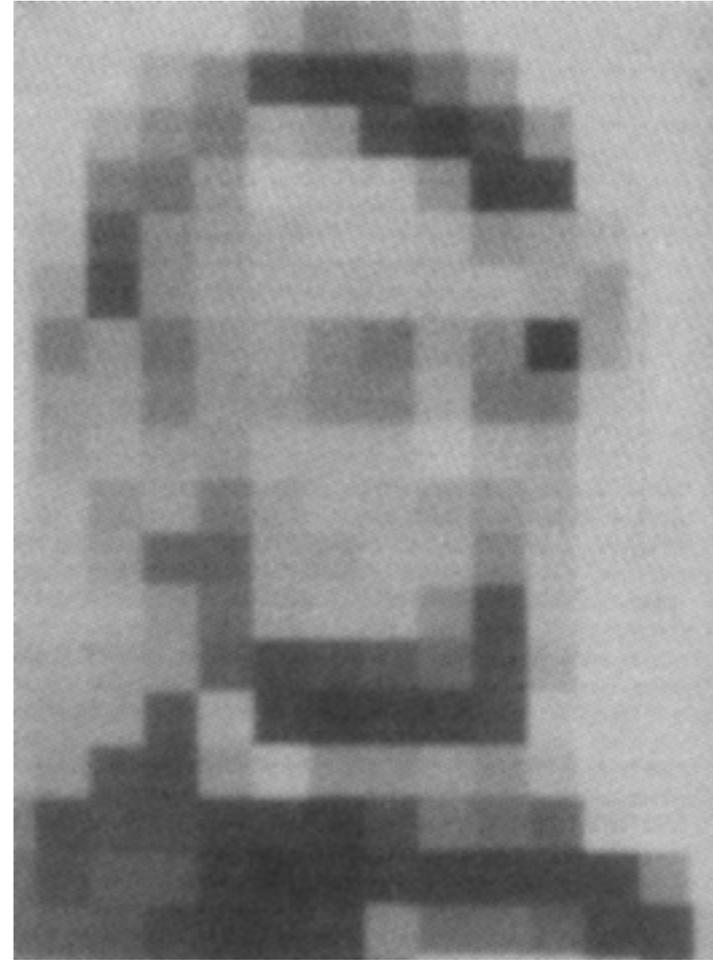
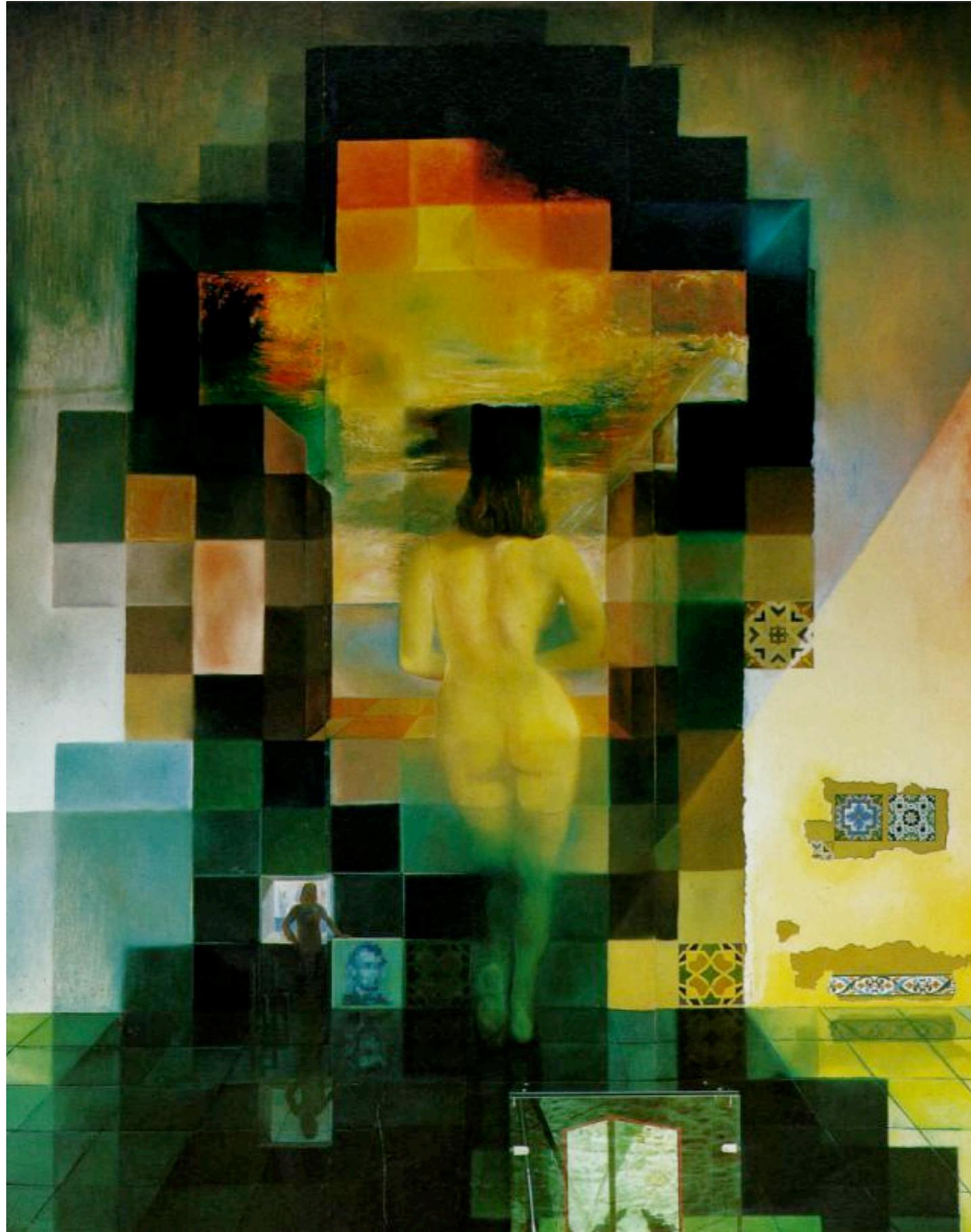
# Weighted Area Sampling



# Gupta-Sproull-Algorithmus

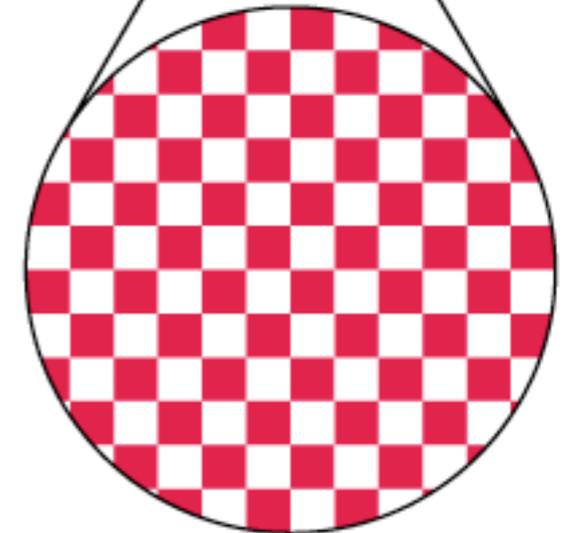
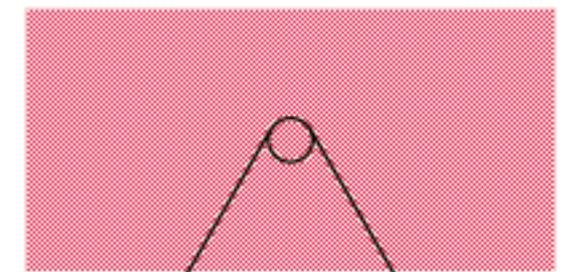
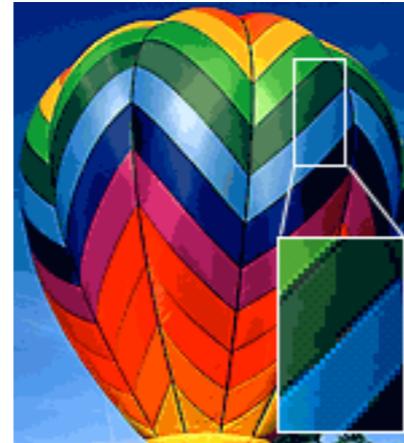
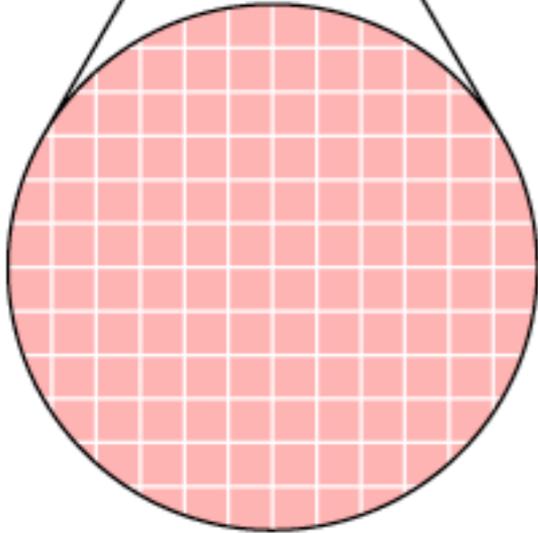
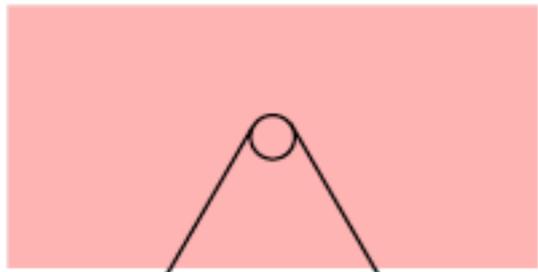


Abstand	Überdeckung	Stufe
0.7 ... 1.0	0-1/8	0 (schwarz)
0.6 ... 0.7	1/8-2/8	1
0.54 ... 0.6	2/8-3/8	2
0.5 ... 0.54	3/8-4/8	3
0.46 ... 0.5	4/8-5/8	4
0.4 ... 0.46	5/8-6/8	5
0.3 ... 0.4	6/8-7/8	6
0 ... 0.3	7/8-1	7 (weiß)



# Dithering

Reduziert Quantisierungsfehler



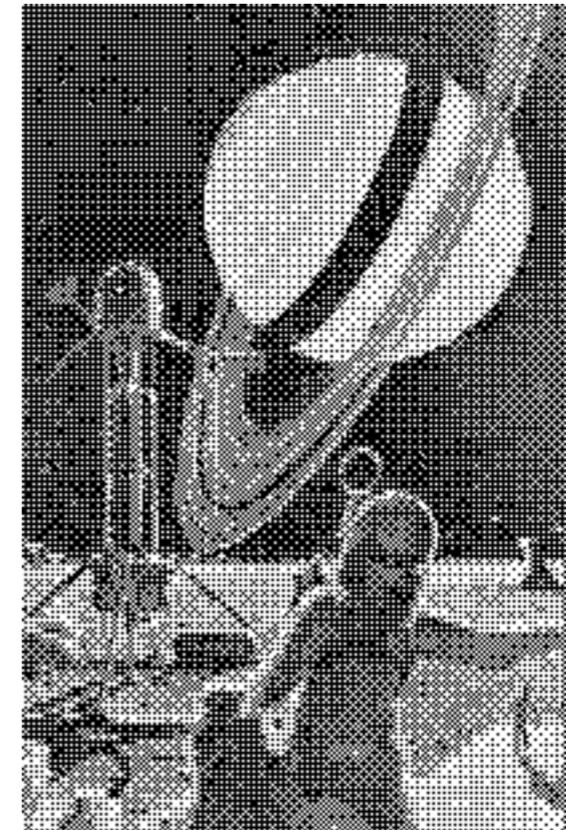
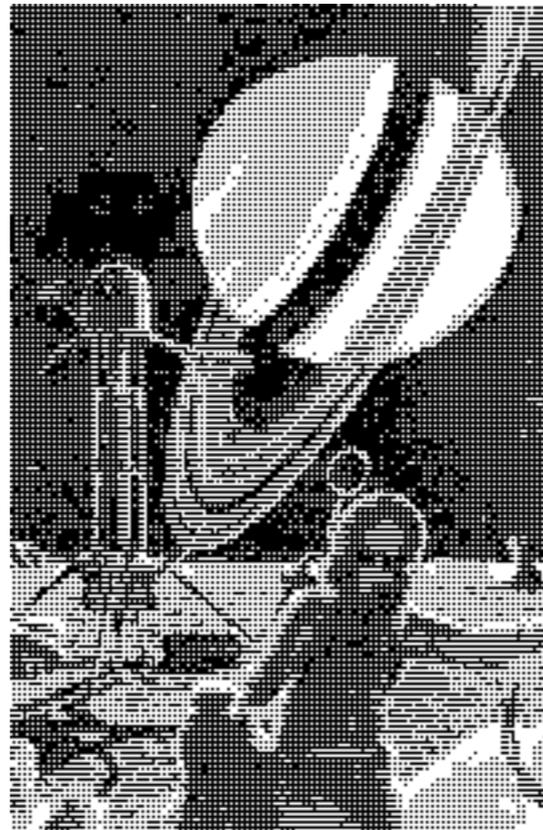
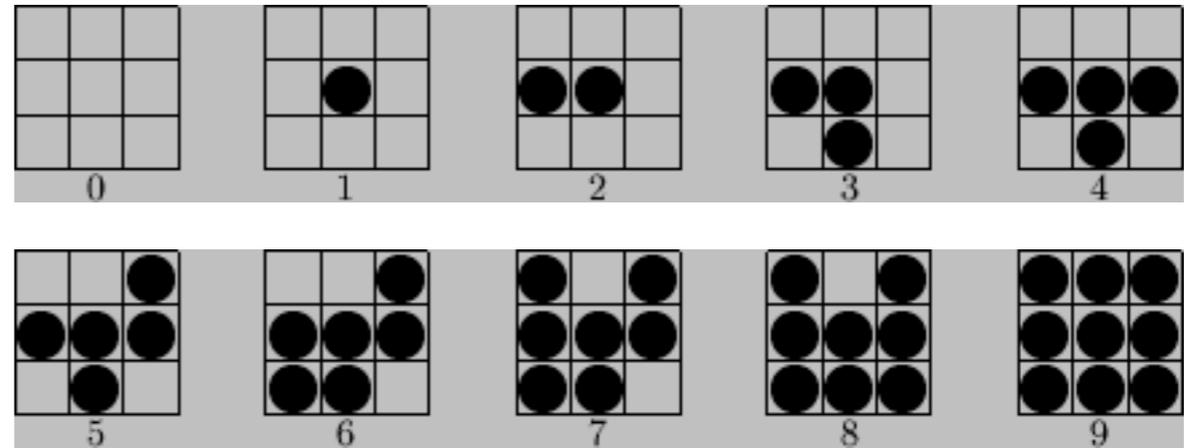
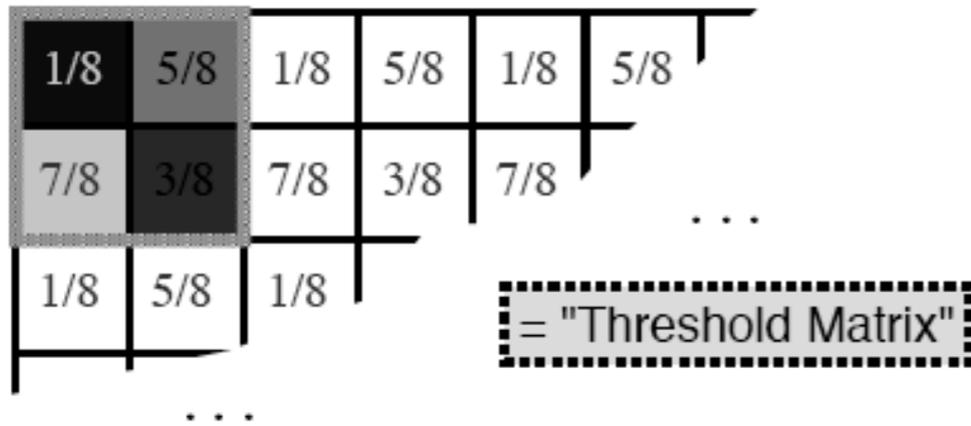
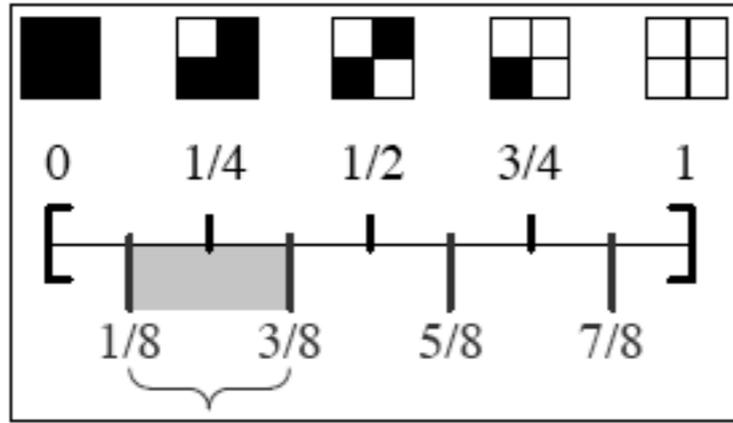
# Constant Threshold Dithering

sample image    threshold values    result

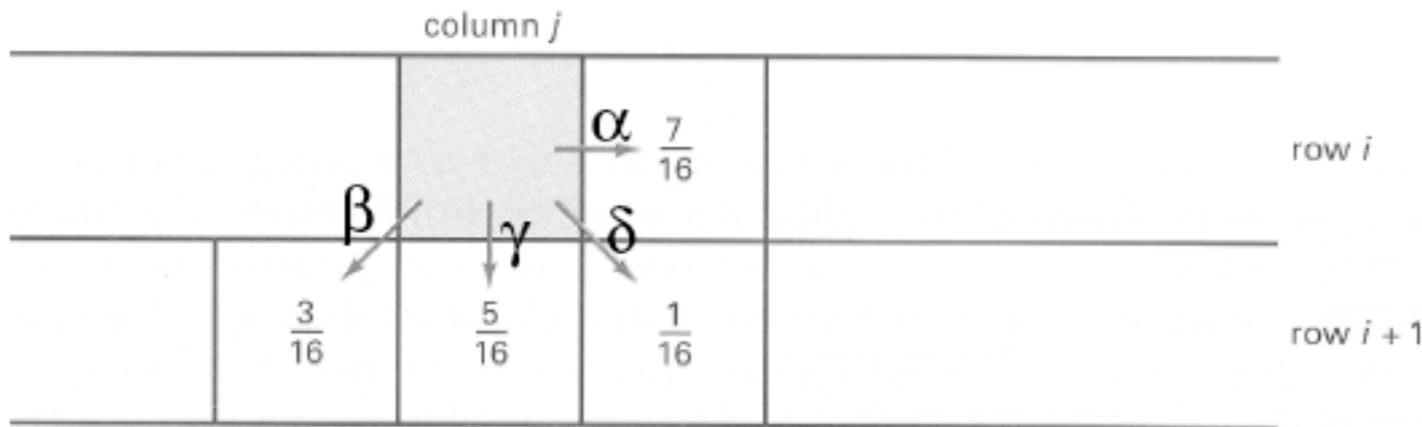
1	7	6	5	4.5	4.5	4.5	4.5	0	9	9	9
1	6	5	4	4.5	4.5	4.5	4.5	0	9	9	0
1	5	4	3	4.5	4.5	4.5	4.5	0	9	0	0
1	4	2	1	4.5	4.5	4.5	4.5	0	0	0	0



# Clustered-Dot Order Dither



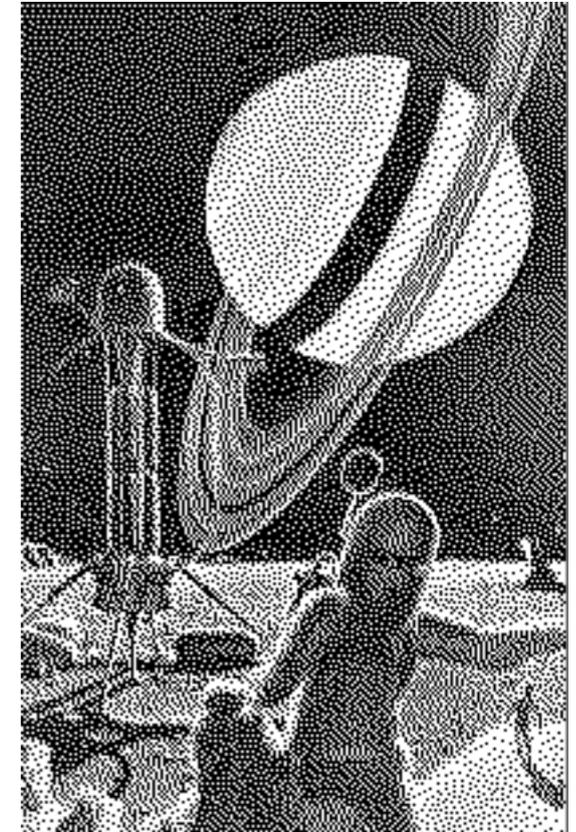
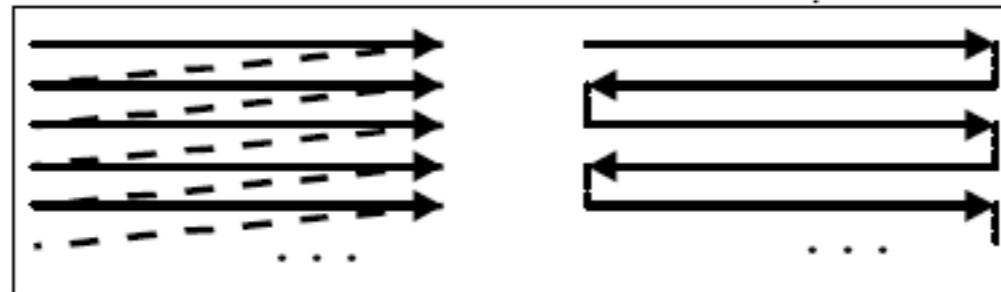
# Floyd-Steinberg-Dither



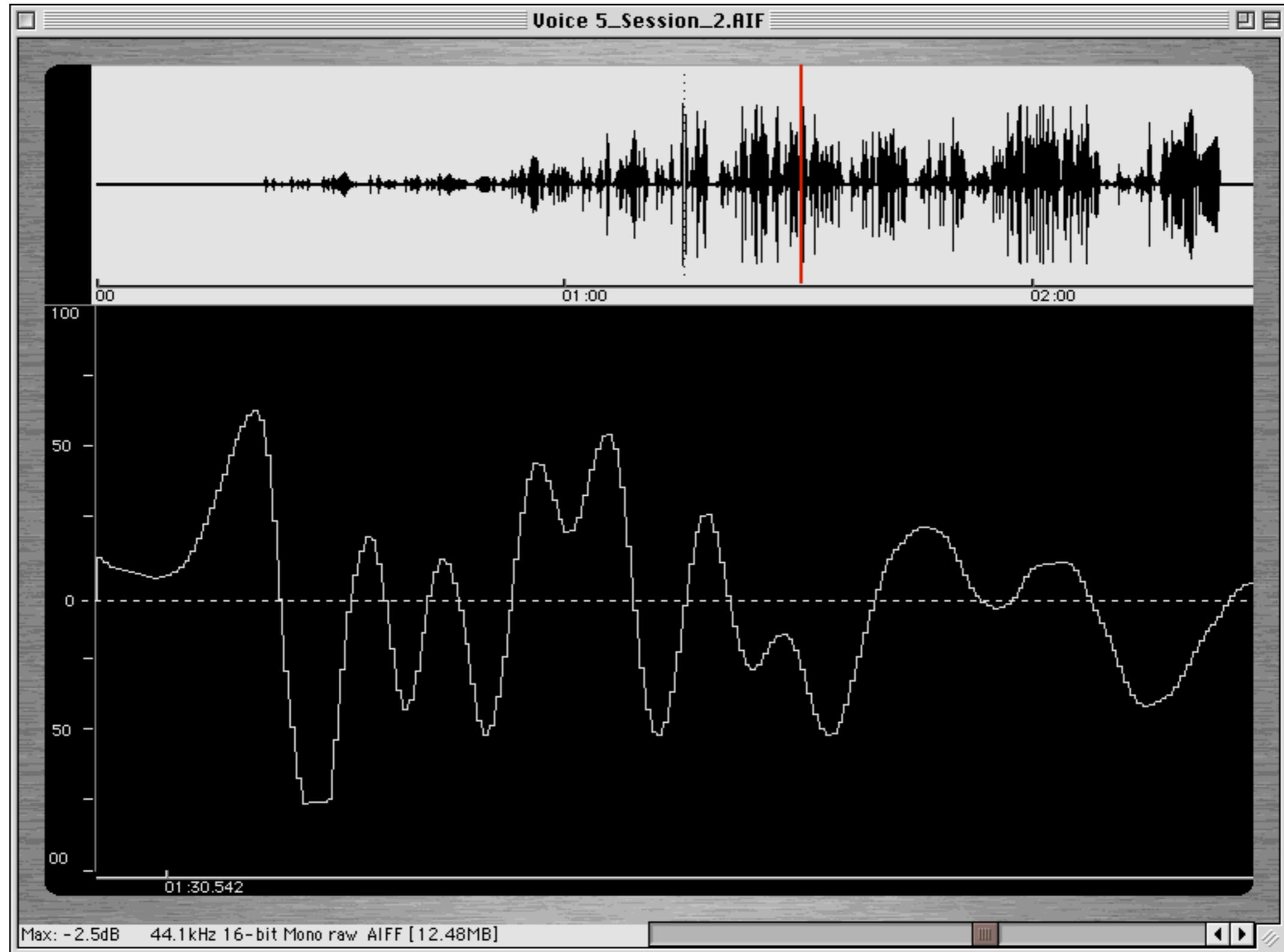
$$\alpha + \beta + \gamma + \delta = 1.0$$

```

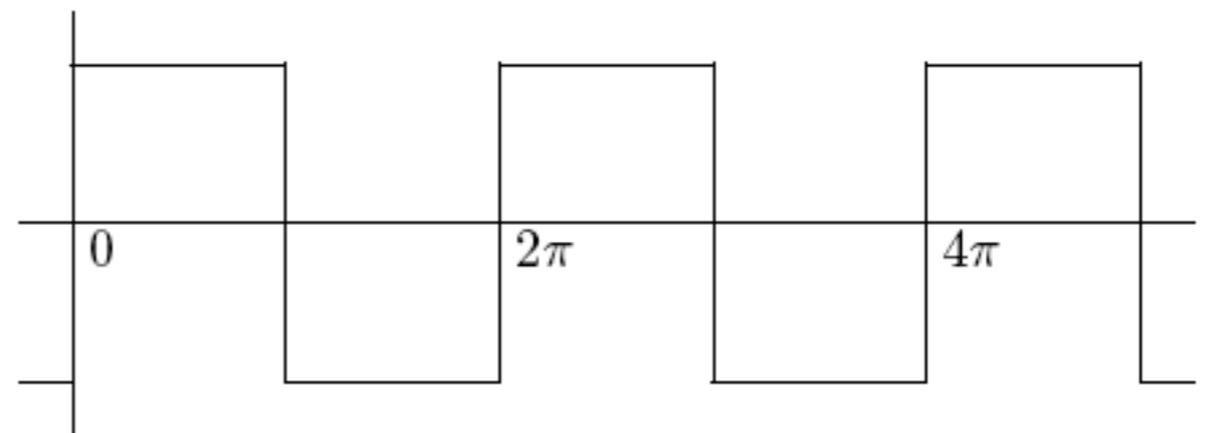
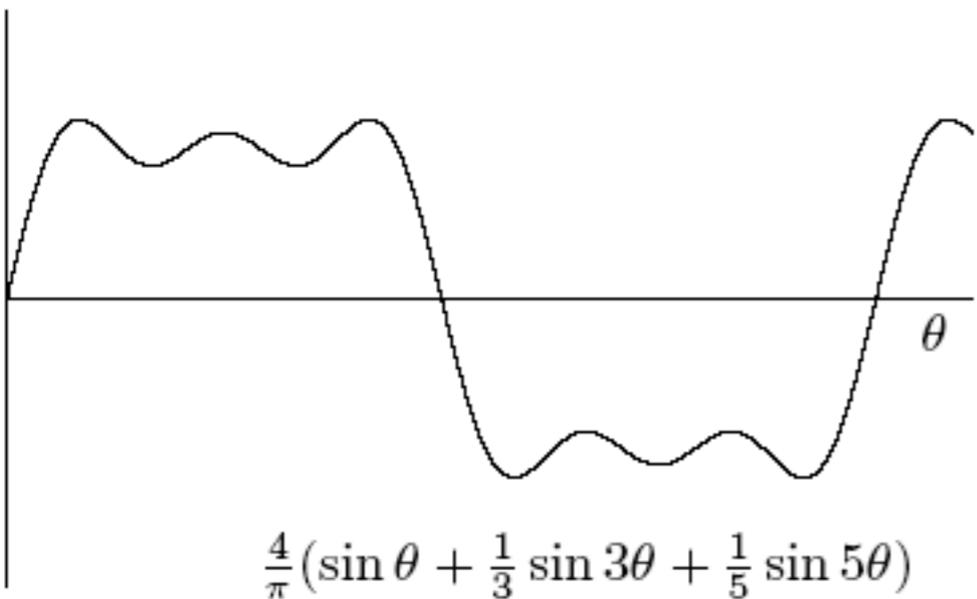
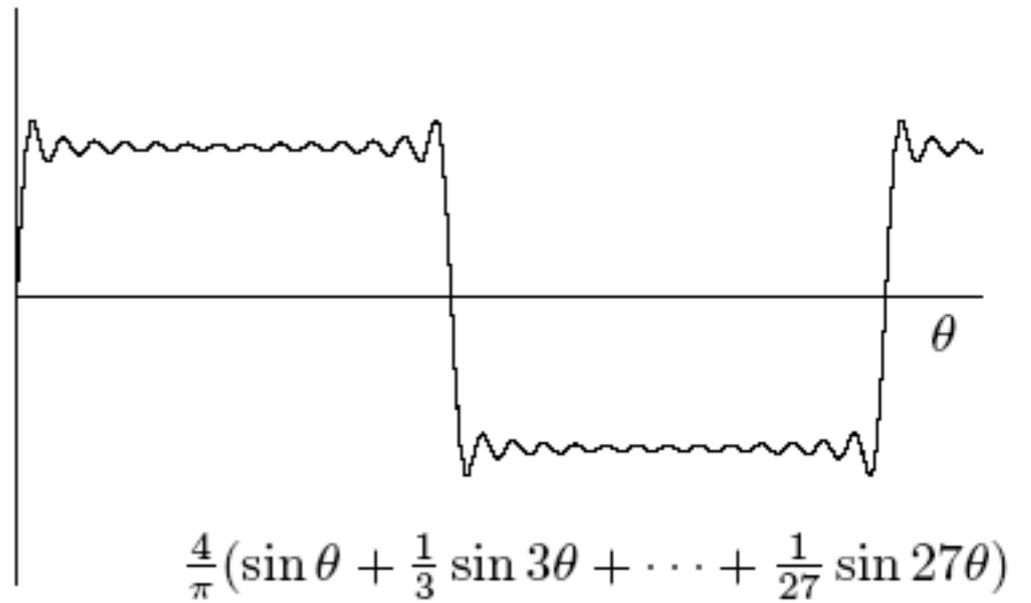
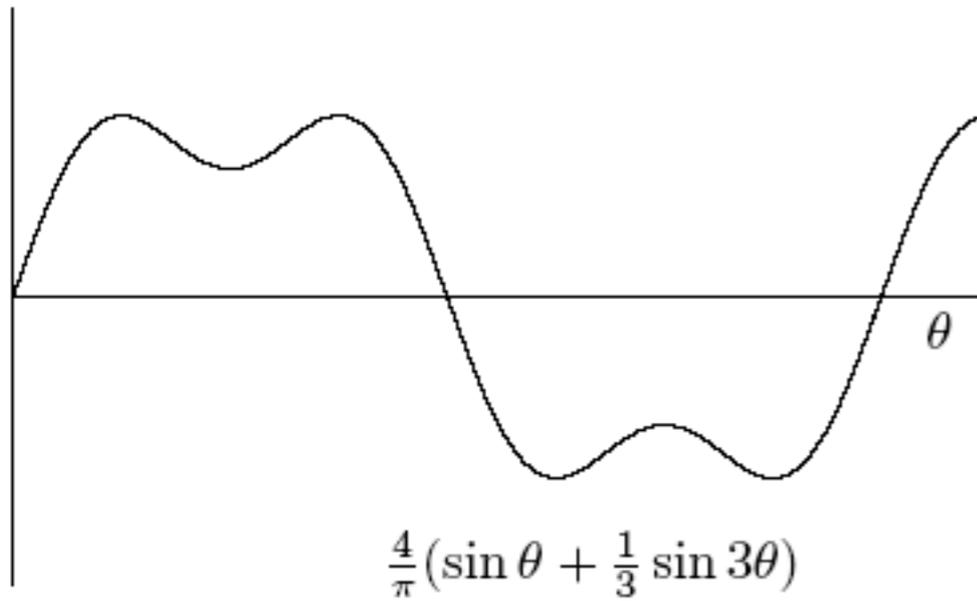
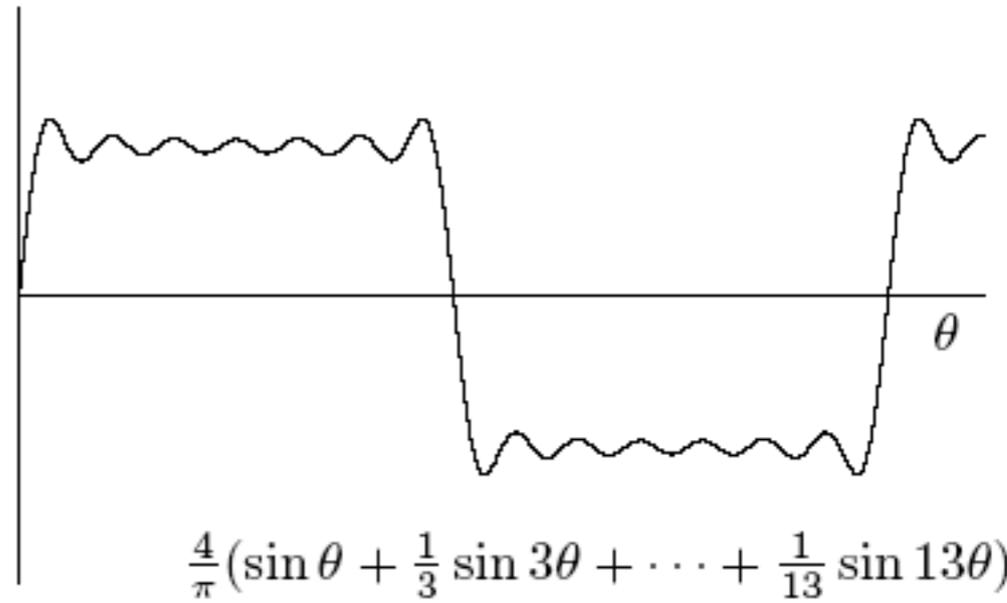
for (x = 0; x < width; x++) {
    for (y = 0; y < height; y++) {
        P(x,y) = trunc(I(x,y) + 0.5)
        e = I(x,y) - P(x,y)
        I(x,y+1) +=  $\alpha$ *e;
        I(x+1,y-1) +=  $\beta$ *e;
        I(x+1,y) +=  $\gamma$ *e;
        I(x+1,y+1) +=  $\delta$ *e;
    }
}
    
```



# Audio

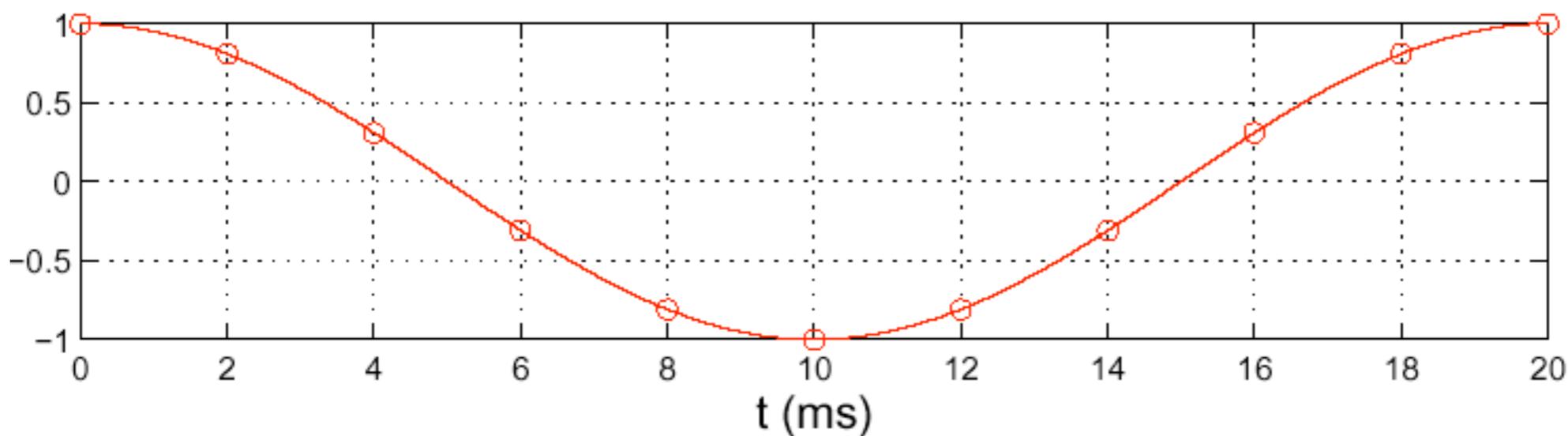
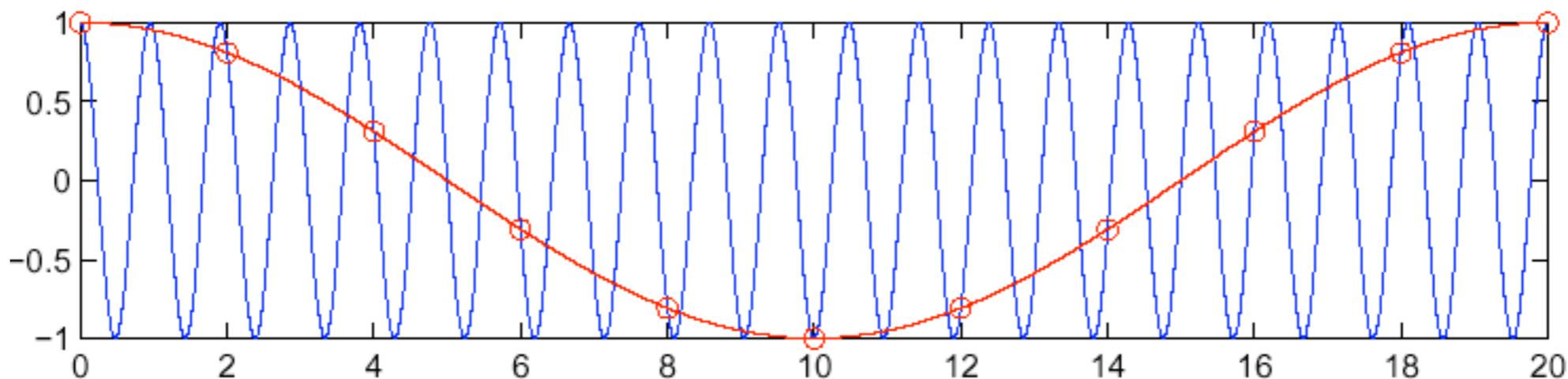
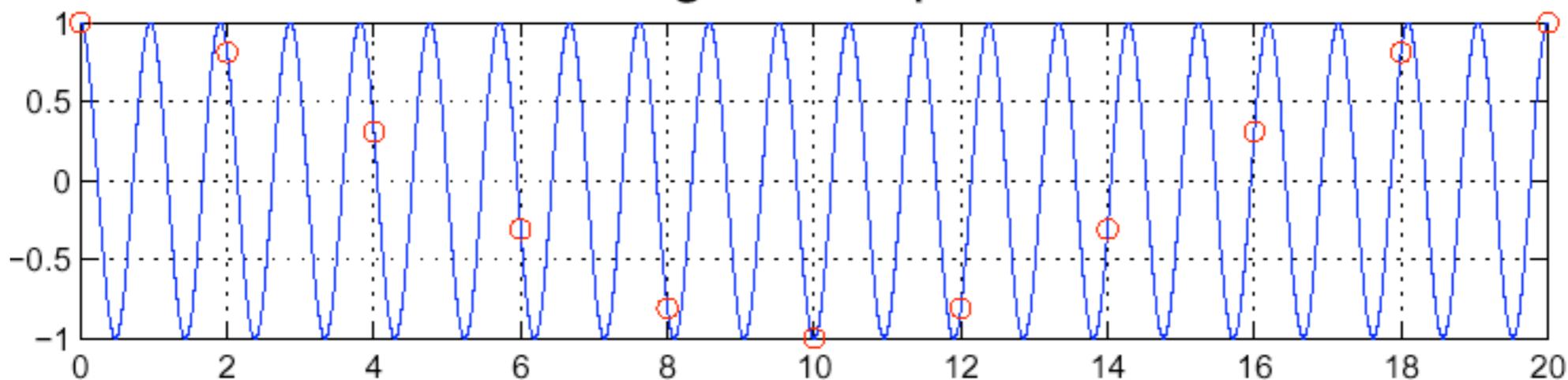


# Fourieranalyse

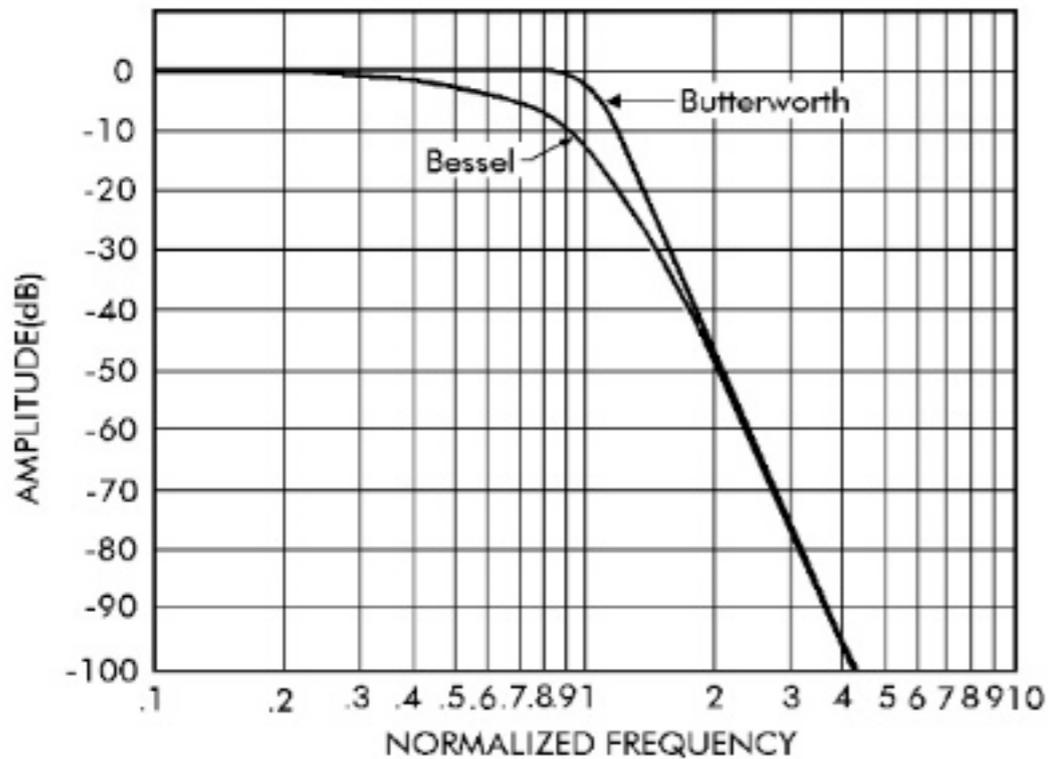


# Nyquisttheorem

A 1050 Hz signal sampled at 500 Hz



# Anti-Aliasfilter



## Tiefpassfilter

- vernachlässigbares Überschwingen (Ringing)
- geringer Phasengang im Durchlaßbereich
- geringste Gruppenlaufzeit-Verzerrungen
- niedriger Klirrfaktor
- hohe Sperrdämpfung
- hohe Steilheit oberhalb  $f_{max}$
- Eigenstörspannung noch unterhalb des Rauschens des digitalen Systems selbst

# Frequenz

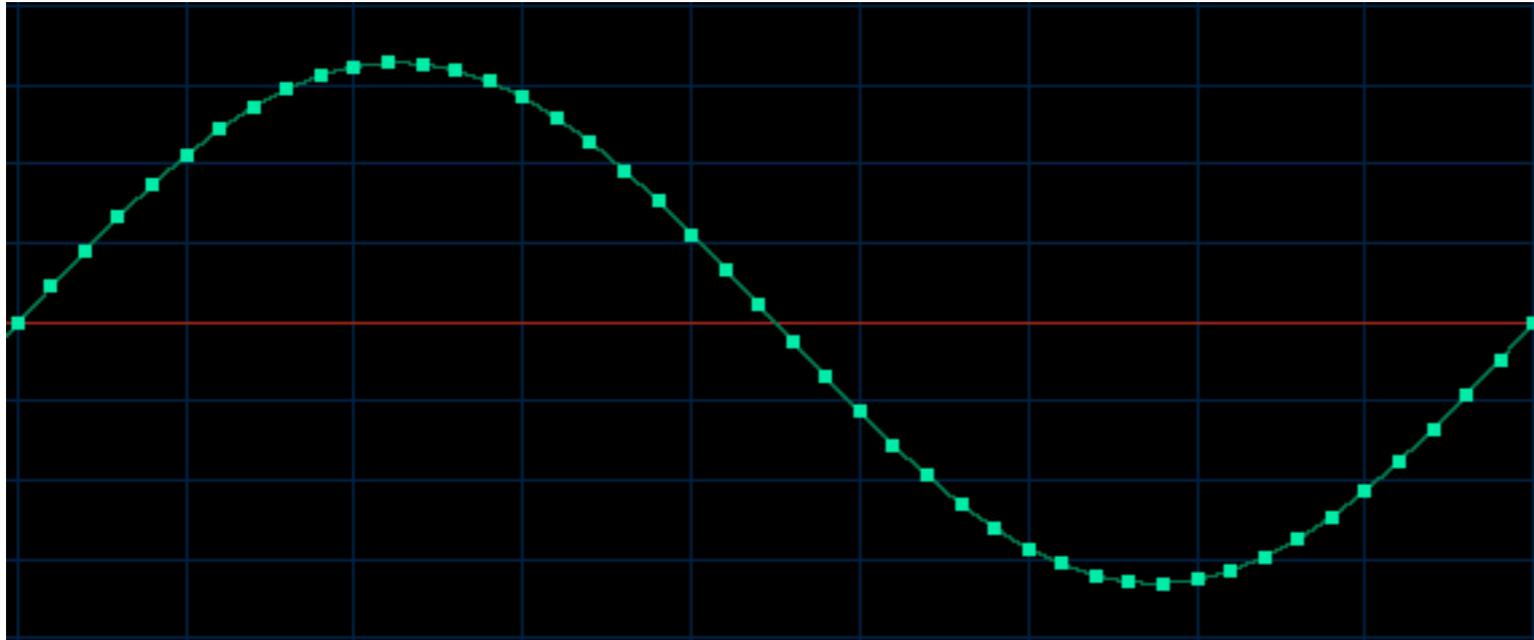


Figure 1 - 980Hz sine wave, -60dB, 24 bits, 44.1kHz

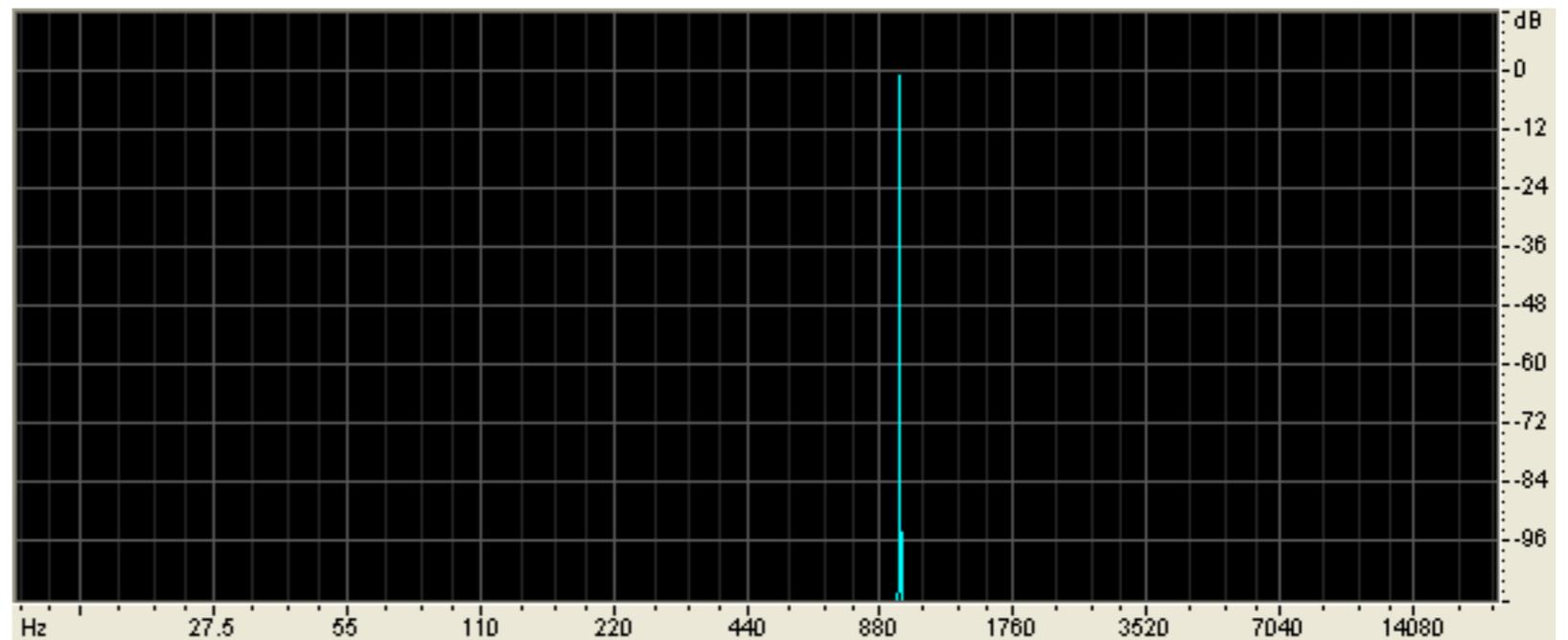


Figure 2 - frequency spectrum for 980Hz sine wave, -60dB, 24 bits, 44.1kHz

# Downsampling

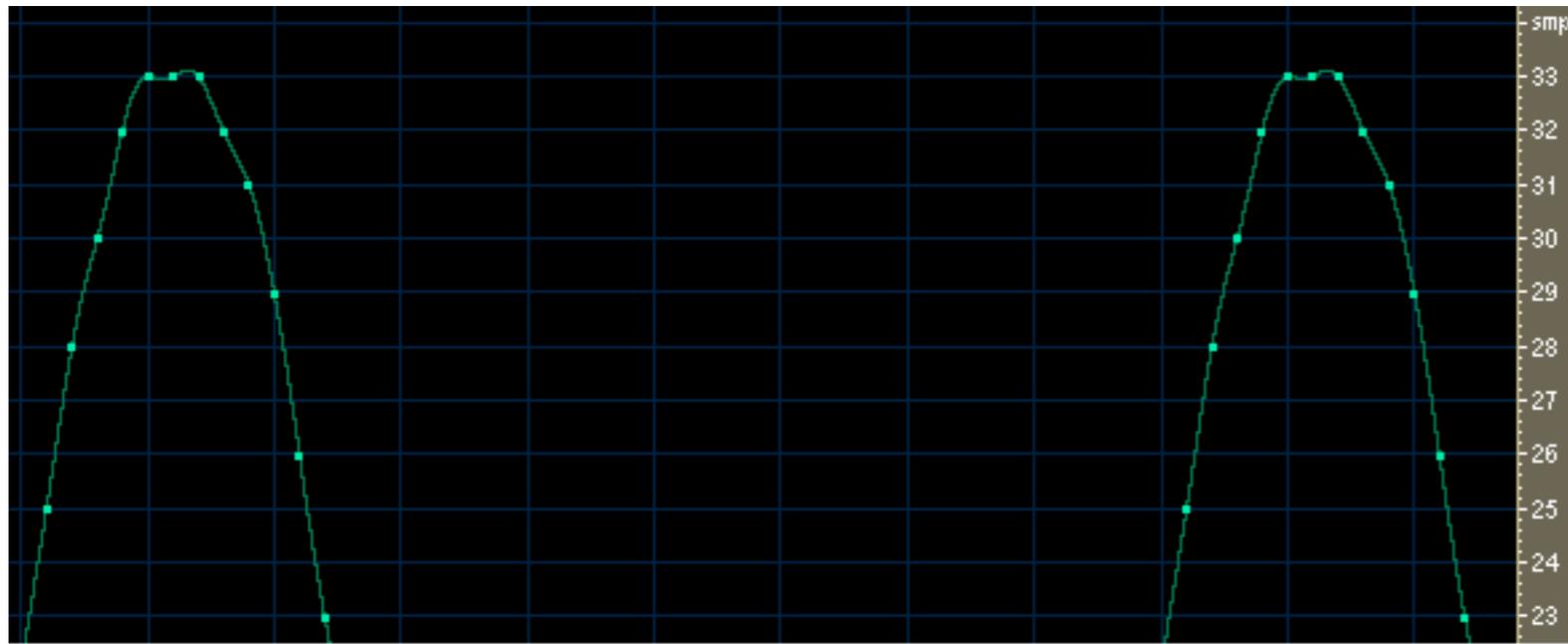


Figure 4 - 980Hz sine wave, -60dB, 16 bits, 44.1kHz

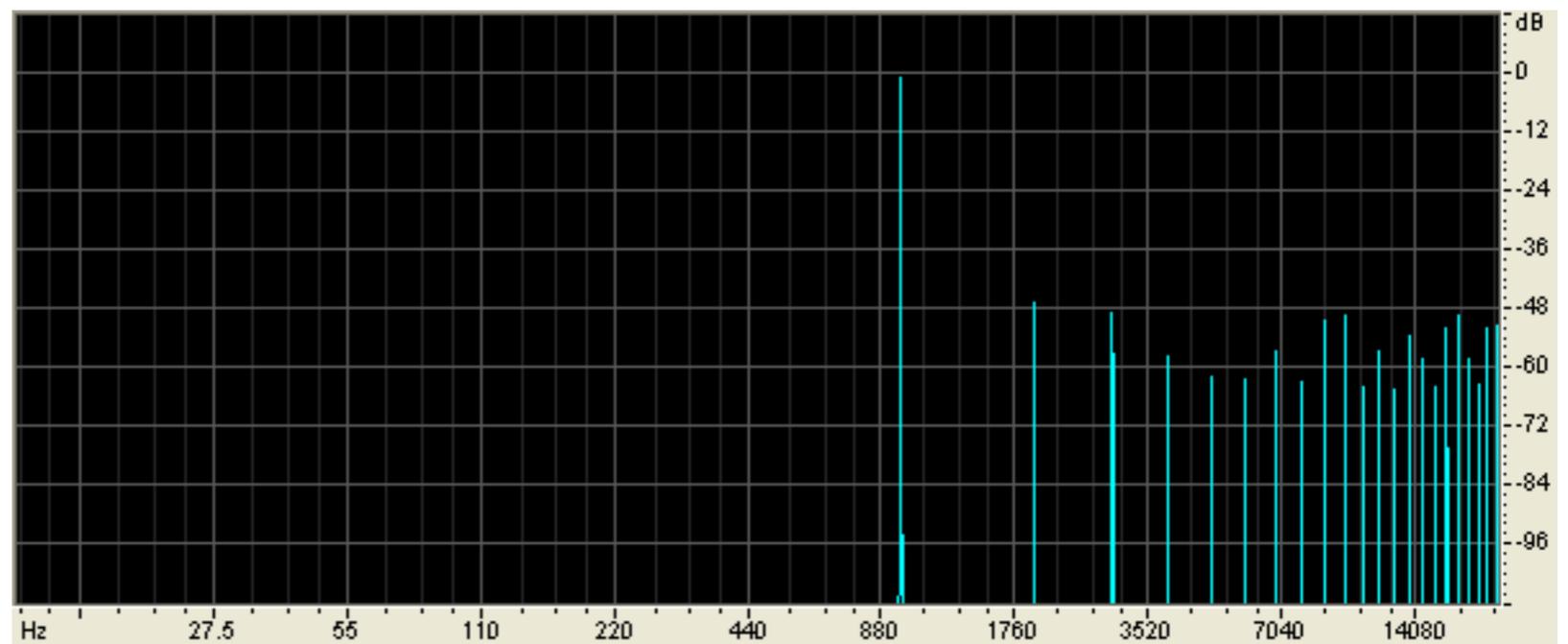


Figure 3 - frequency spectrum for 980Hz sine wave, -60dB, 16 bits, 44.1kHz

# Dithering

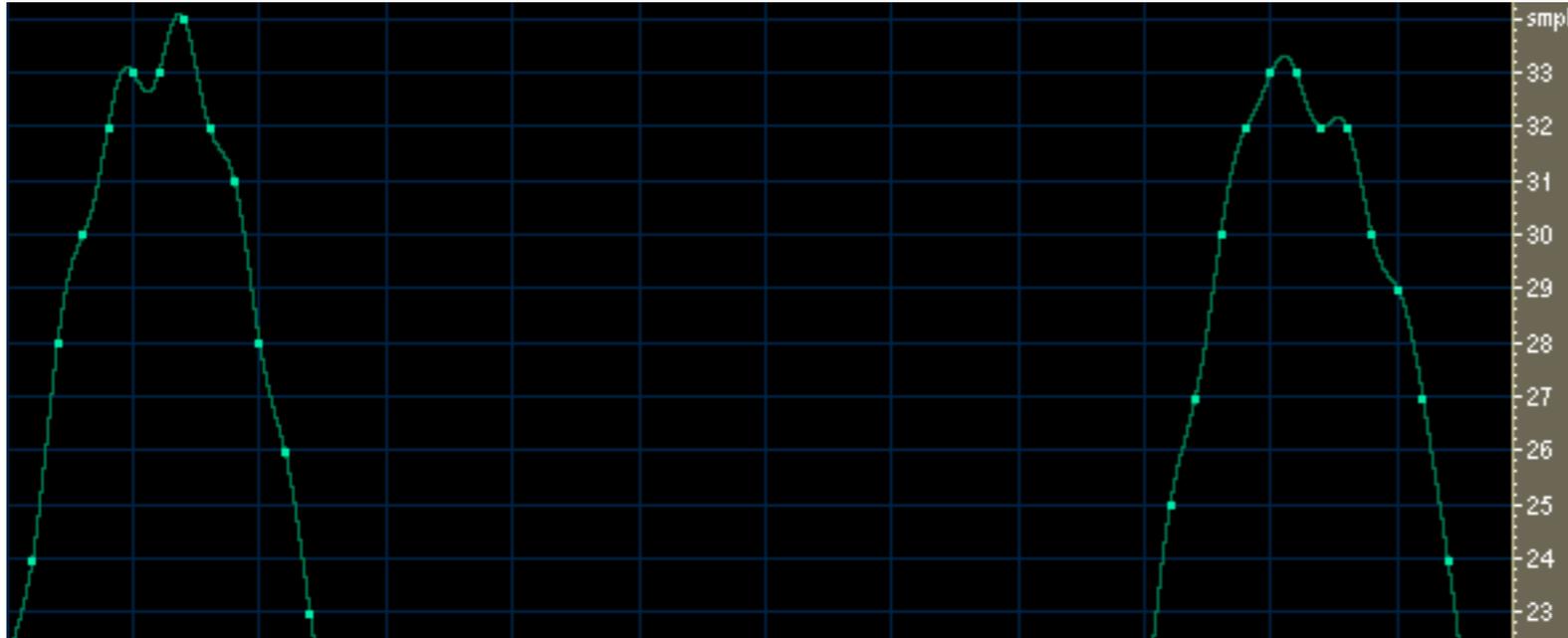


Figure 6 - 980Hz sine wave, -60dB, 16 bits, 44.1kHz, dithered

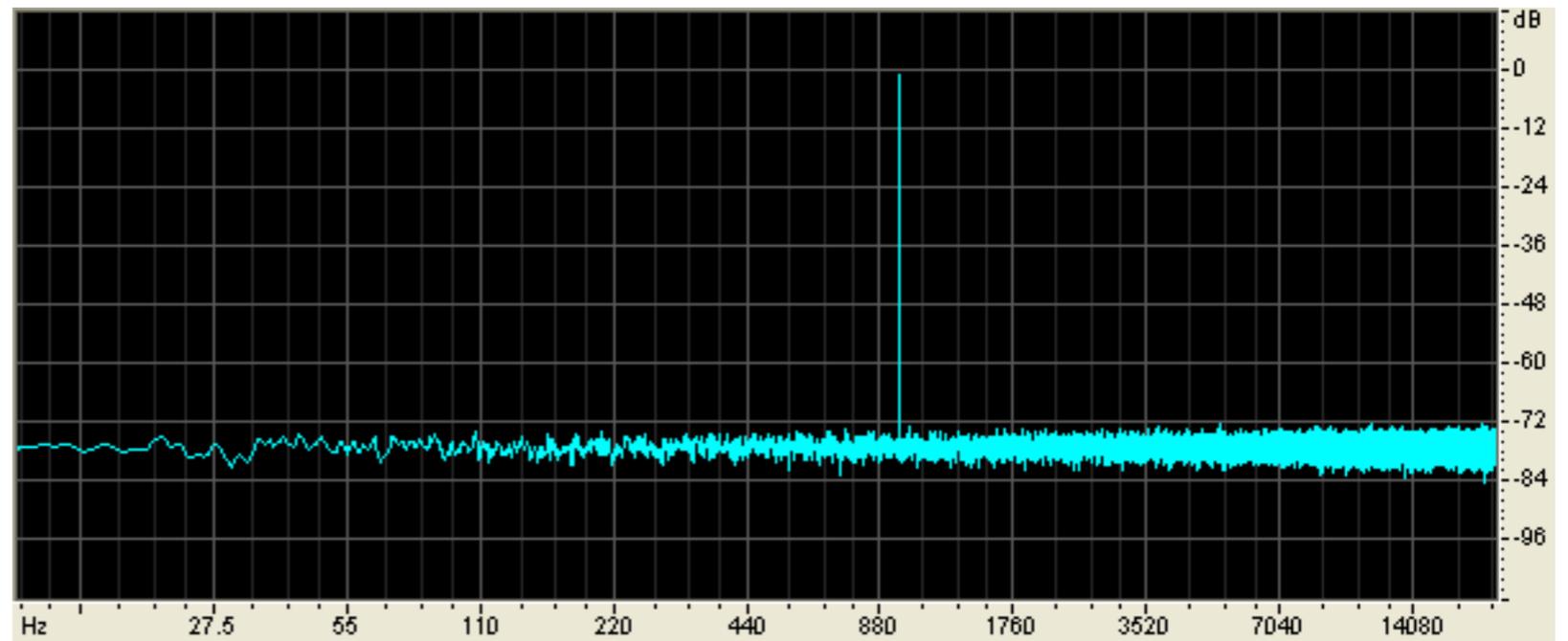


Figure 5 - frequency spectrum for 980Hz sine wave, -60dB, 16 bits, 44.1kHz, with dithering

# Noise Shaping

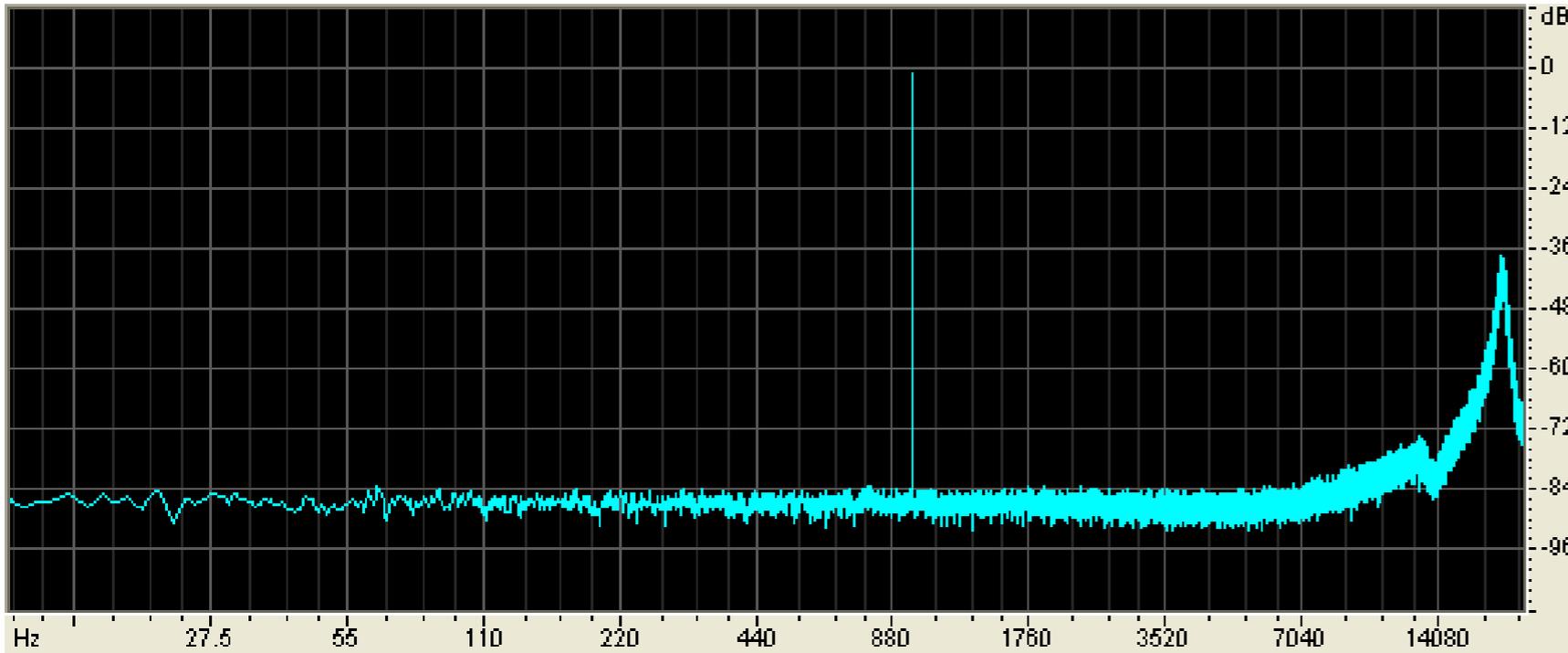
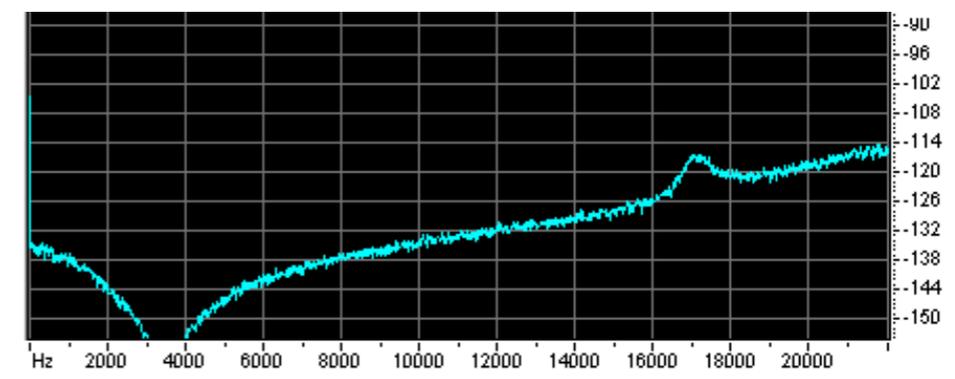
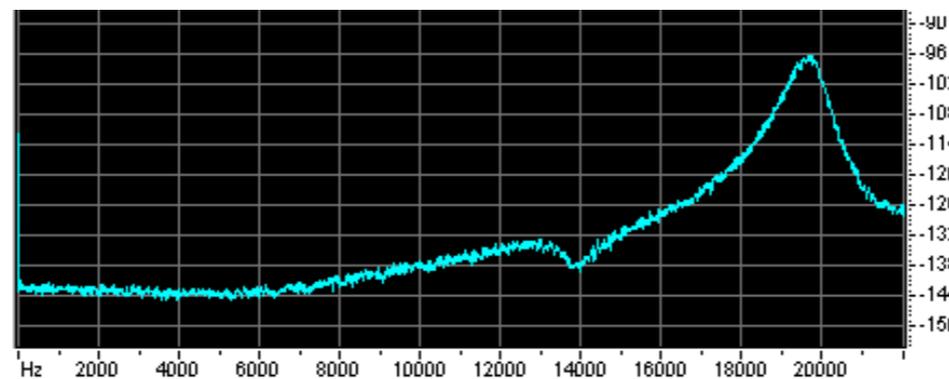
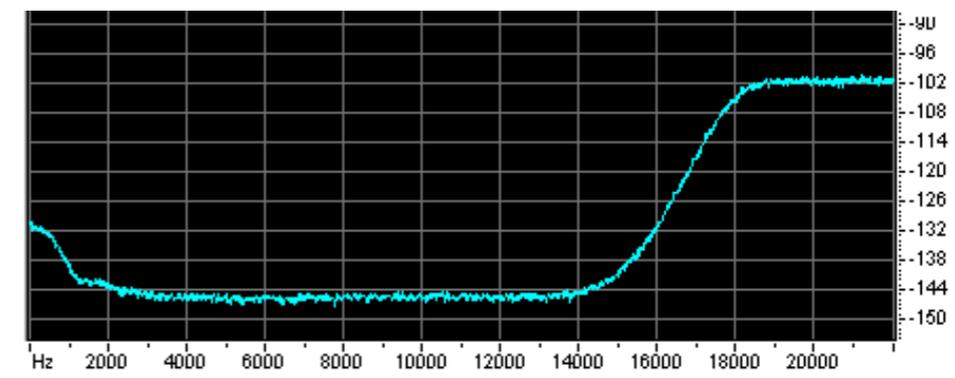
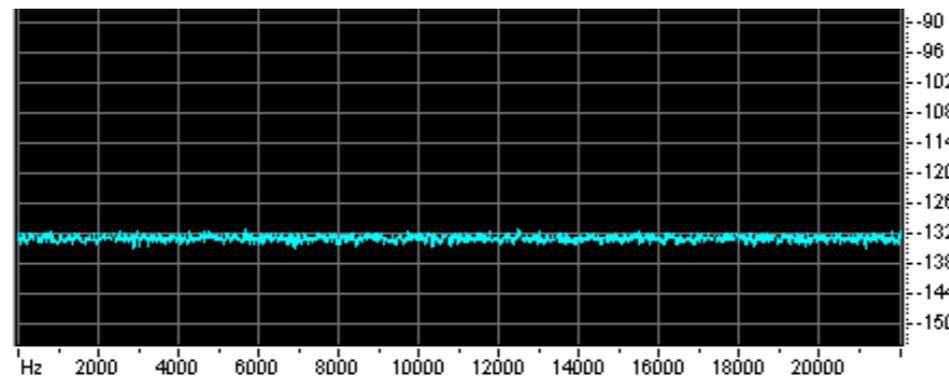
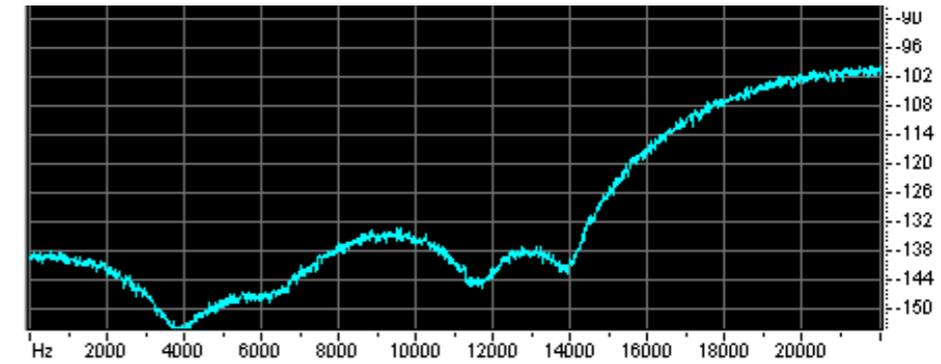
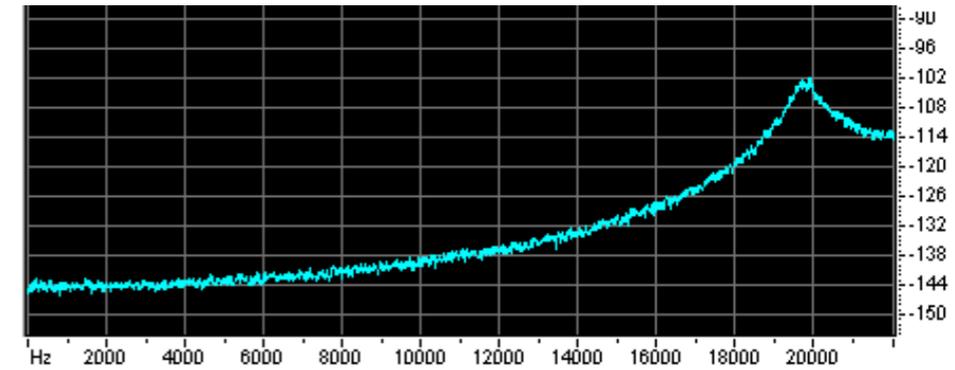
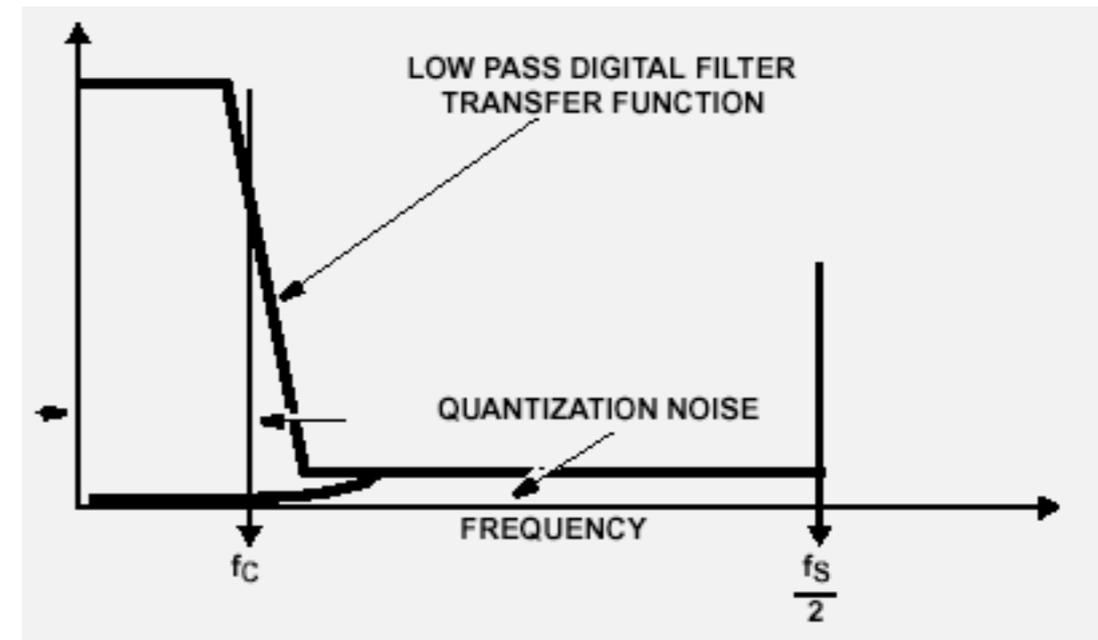
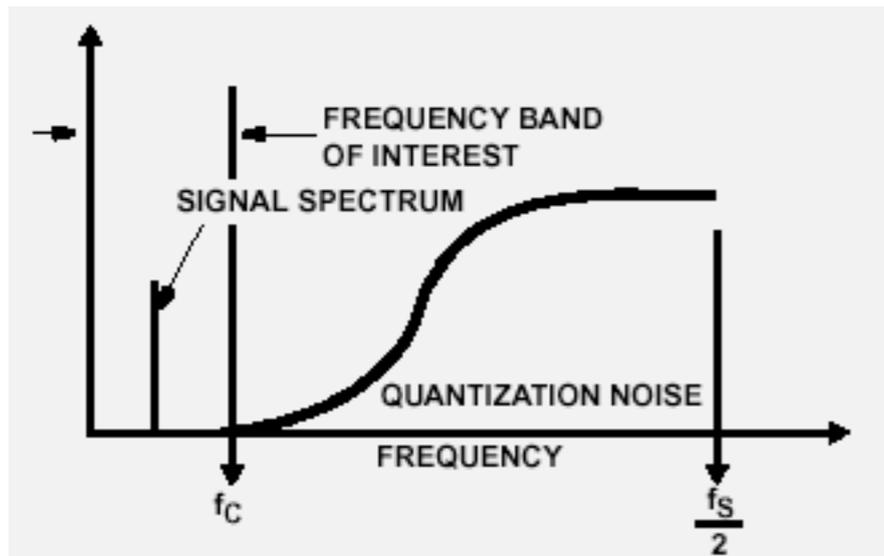
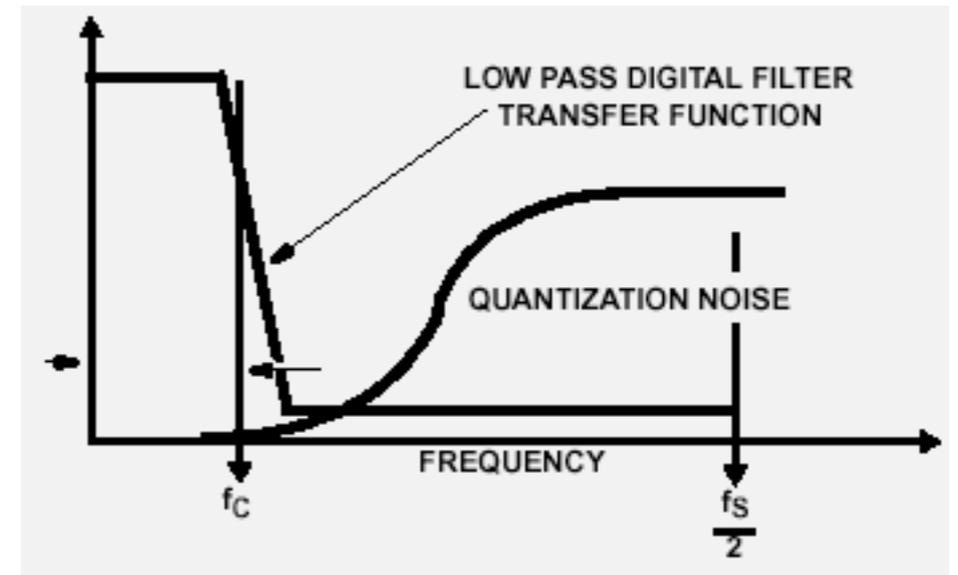
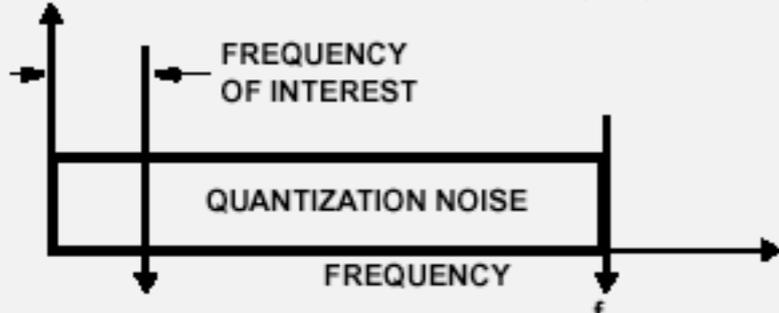


Figure 7 - frequency spectrum for 980Hz sine wave, -60dB, 24 bits, 44.1kHz, dithering with noise shaping

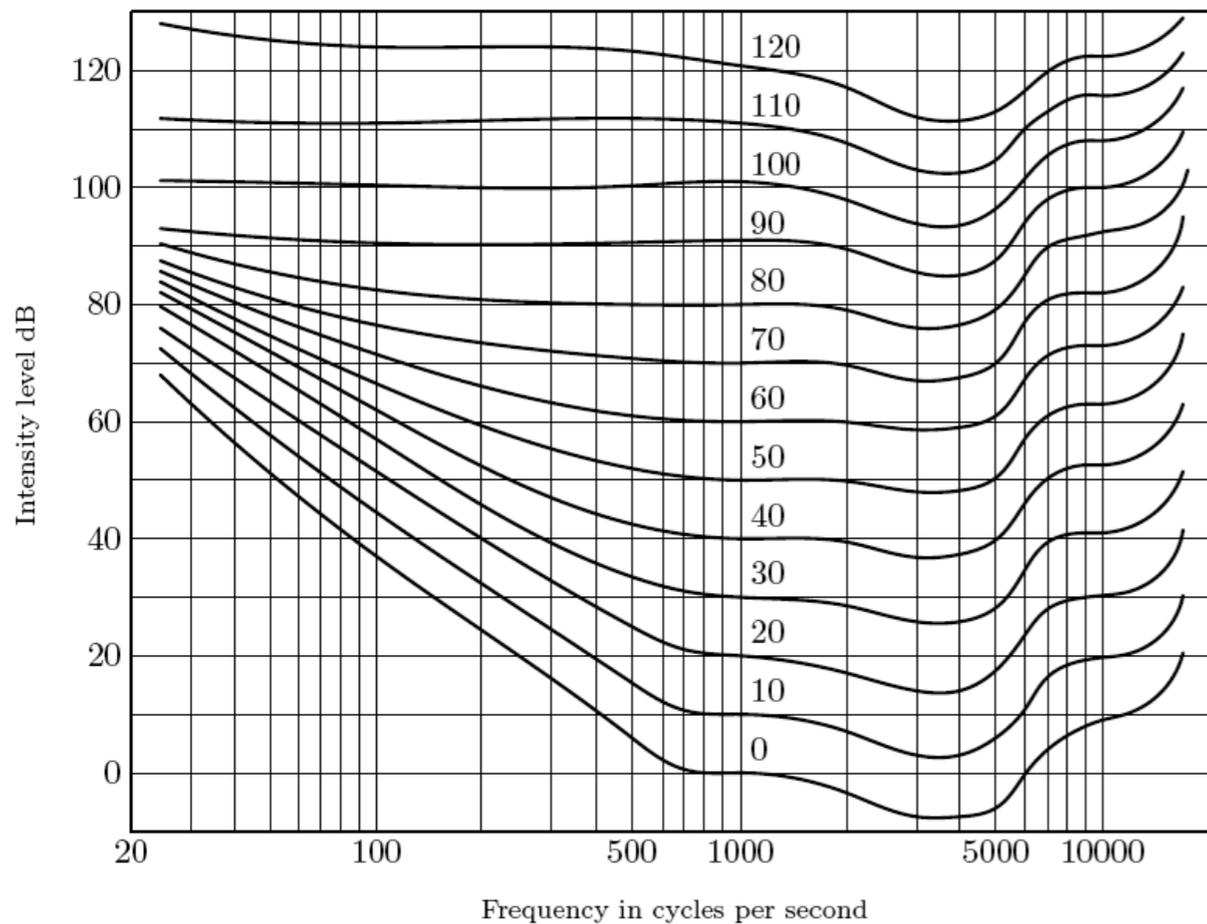
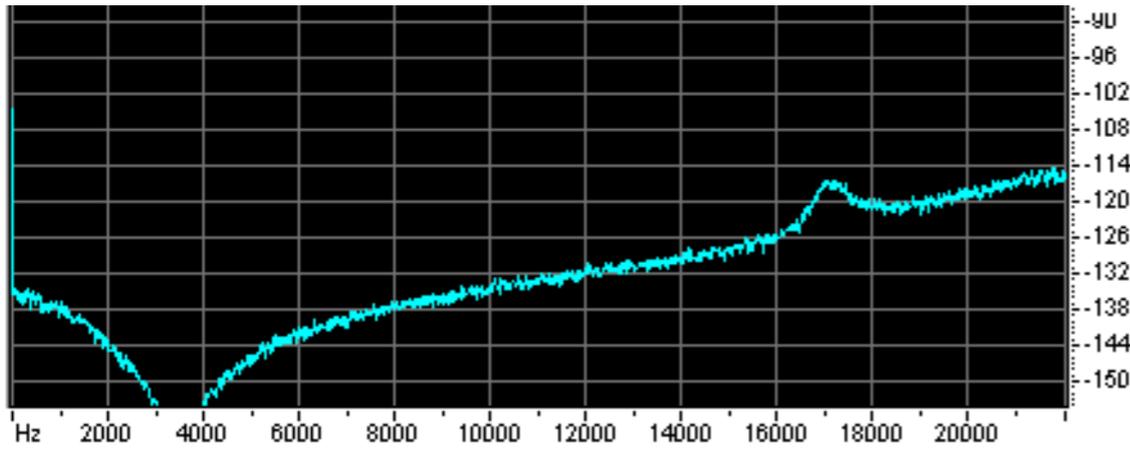


# Noise Shaping

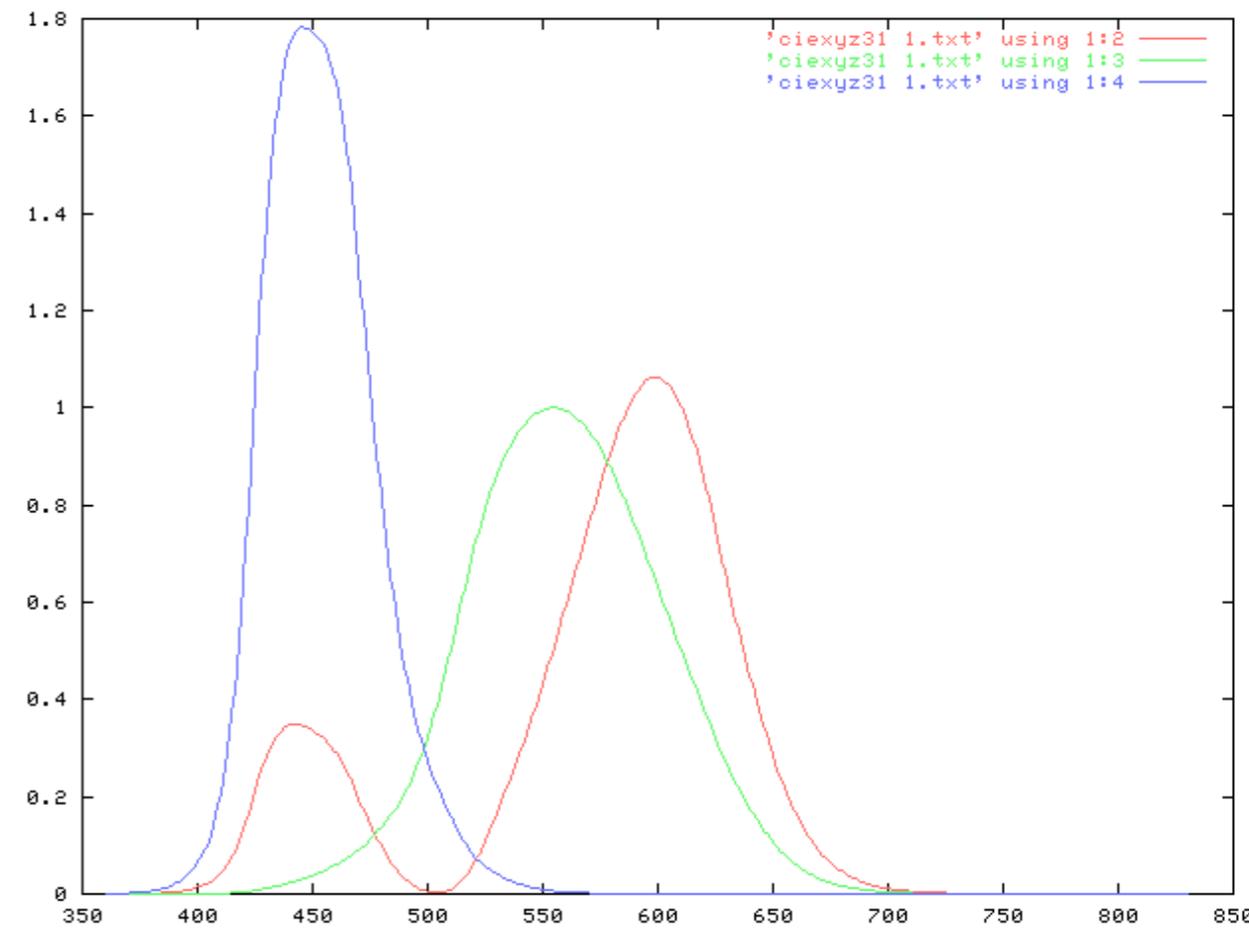
$$\text{SNR} = (6.02N + 1.76) \text{dB} + 10 \text{LOG}_{10} \left( \frac{F_s}{2F_c} \right) \text{dB}$$



# Psychophysische Modelle



Fletcher-Munson-Kurven



CIE 1931

Die Grenze zwischen  
Analog und Digital wird  
von Menschen für  
Menschen gezogen.